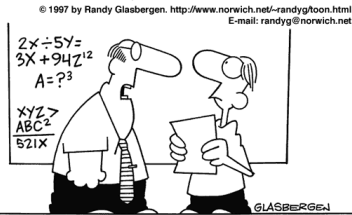
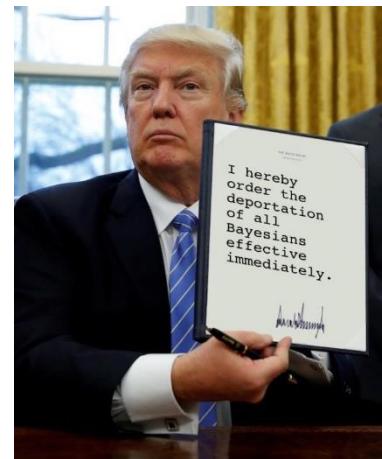
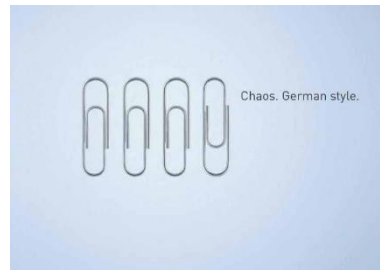


$$x^3 - 6'156x^2 + 12'631'328x - 8'638'755'840 = 0$$



“Why is it important for today’s kids to learn algebra? Because I had to learn this junk in school and now it’s your turn, that’s why!”



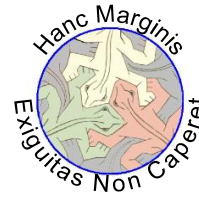
Frank and Ernest



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1	1	M	(1803) Guglielmo Libri Carucci dalla Sommaja	RM132
			(1878) Agner Krarup Erlang	
	2	T	(1822) Rudolf Julius Emmanuel Clausius	RM240
			(1905) Lev Genrichovich Shnirelman	
			(1938) Anatoly Samoilenko	
	3	W	(1777) Louis Poinso	
			(1917) Yuri Alexeievich Mitropolsky	
4	T	(1643) Isaac Newton	RM071	
		(1723) Nicole-Reine Étable de Labrière Lepaute	RM288	
5	F	(1838) Marie Ennemond Camille Jordan	RM276	
		(1871) Federigo Enriques	RM084	
		(1871) Gino Fano		
6	S	(1807) Jozeph Mitza Petzval		
		(1841) Rudolf Sturm		
7	S	(1871) Felix Edouard Justin Émile Borel		
		(1907) Raymond Edward Alan Christopher Paley		
1925	Walter Noll			
2	8	M	(1888) Richard Courant	RM156
			(1924) Paul Moritz Cohn	
			(1942) Stephen William Hawking	
	9	T	(1864) Vladimir Adreievich Steklov	RM252
			(1882) Pavel Aleksandrovitch Florenskij	
	10	W	(1915) Mollie Orshansky	
			(1875) Issai Schur	
	11	T	(1905) Ruth Moufang	
			(1545) Guidobaldo del Monte	RM120
	12	F	(1707) Vincenzo Riccati	
			(1734) Achille Pierre Dionis du Séjour	
	13	S	(1853) Gregorio Ricci-Curbastro	
			(1906) Kurt August Hirsch	
			(1915) Herbert Ellis Robbins	RM156
14	S	(1864) Wilhelm Karl Werner Otto Fritz Franz Wien		
		(1876) Luther Pfahler Eisenhart		
15	M	(1876) Erhard Schmidt		
		(1902) Karl Menger		
16	T	(1901) Alfred Tarski	RM096	
		(1704) Johann Castillon		
17	W	(1717) Mattew Stewart		
		(1850) Sofia Vasilievna Kovalevskaya	RM144	
18	T	(1801) Thomas Klausen		
		(1647) Catherina Elisabetha Koopman Hevelius	RM264	
19	F	(1847) Nikolay Egorovich Zukowsky		
		(1858) Gabriel Koenigs		
20	S	(1856) Luigi Bianchi	RM204	
		(1880) Paul Ehrenfest		
21	S	(1813) Rudolf Friedrich Alfred Clebsch		
		(1879) Guido Fubini		
22	M	(1908) Aleksandr Gennadievich Kurosh		
		(1775) André Marie Ampère		
23	T	(1895) Gabor Szegő	RM072	
		(1904) Renato Caccioppoli		
24	W	(1846) Pieter Hendrik Schoute	RM252	
		(1882) Pavel Aleksandrovitch Florenskij		
25	T	(1915) Yuri Vladimirovich Linnik		
		(1561) Francis Bacon		
26	F	(1592) Pierre Gassendi	RM228	
		(1886) John William Navin Sullivan		
27	S	(1908) Lev Davidovich Landau		
		(1840) Ernst Abbe	RM060	
28	S	(1862) David Hilbert		
		(1891) Abram Samoilovitch Besicovitch		
29	M	(1902) Oskar Morgenstern		
		(1914) Vladimir Petrovich Potapov		
30	T	(1627) Robert Boyle	RM048	
		(1736) Joseph-Louis Lagrange		
31	W	(1843) Karl Hermann Amandus Schwarz		
		(1799) Benoît Paul Émile Clapeyron		
32	F	(1849) Francesco Flores D'Arcais		
		(1862) Eliakim Hastings Moore		
33	S	(1832) Charles Lutwidge Dodgson	RM108	
		(1611) Johannes Hevelius	RM264	
34	S	(1701) Charles Marie de La Condamine		
		(1888) Louis Joel Mordell		
35	M	(1892) Carlo Emilio Bonferroni		
		(1817) William Ferrel		
36	T	(1888) Sidney Chapman		
		(1619) Michelangelo Ricci	RM216	
37	W	(1715) Giovanni Francesco Fagnano dei Toschi		
		(1841) Samuel Loyd	RM192	
38	W	(1896) Sofia Alexandrovna Janowskaja		

Putnam 2009, A1

Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

How to put an elephant into a refrigerator

Analysis

Method 1: Differentiate the elephant, put it in the refrigerator, and then integrate it into the refrigerator domain.

Method 2: Redefine the refrigerator metric.

Method 3: Apply the Banach-Tarsky Theorem.

Murphy's Laws applied to Math Students

When you solve a problem, it always helps to know the answer.

For many parts of Nature can neither be invented with sufficient subtlety, nor demonstrated with sufficient perspicuity, nor accommodated unto use with sufficient dexterity, without the aid and intervening of the mathematics, of which sort are perspective, music, astronomy, cosmography, architecture, engineery, and divers others.

Francis Bacon

Treat nature by the cylinder, the sphere, the cone, everything in proper perspective so that each side of an object or a plane is directed towards a central point.

Paul Cézanne

It is therefore with the study of mathematics, and only through it, that one can form a correct and in-depth idea of what a science is.

Auguste Comte

The straight line is not a thing, but our concept of reality. And if we cannot reveal the concrete content of this concept, if the extent of its application is equal to zero, then this concept does not exist.

Pavel Aleksandrovitch Florenskij

Since the mathematical sciences are so vast and varied, it is necessary to limit their practice and teaching, since every human activity is linked to places and people.

David Hilbert

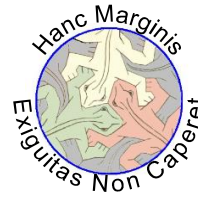
To speak algebraically, Mr. M. is execrable, but Mr. G. is $(x + 1)$ -ecrable.

[Discussing fellow writers Cornelius Mathews and William Ellery Channing.]

Edgar Allan Poe

If a body previously at rest begin to move, we may be certain that this is only in virtue of some extraneous cause acting upon it. This cause, whatever it may be, and which is known to us only by its effects, we call Force. Force then is any cause whatever of motion.

Louis Poinso



1	T	(1900) John Charles Burkill	
2	F	(1522) Lodovico Ferrari (1893) Cornelius Lanczos (1897) Gertrude Blanch	RM229
3	S	(1893) Gaston Maurice Julia	RM073
4	S	(1905) Eric Christopher Zeeman	RM241
6	5 M	(1757) Jean Marie Constant Duhamel	
6	T	(1465) Scipione del Ferro (1612) Antoine Arnauld (1695) Nicolaus (II) Bernoulli	RM064 RM093
7	W	(1877) Godfried Harold Hardy (1883) Eric Temple Bell	RM049
8	T	(1700) Daniel Bernoulli (1875) Francis Ysidro Edgeworth (1928) Ennio de Giorgi	RM093 RM133
9	F	(1775) Farkas Wolfgang Bolyai (1907) Harold Scott Macdonald Coxeter	RM097
10	S	(1747) Aida Yasuaki (1932) Vivienne Malone-Mayes	RM121
11	S	(1657) Bernard Le Bovier de Fontenelle (1800) William Henry Fox Talbot (1839) Josiah Willard Gibbs (1915) Richard Wesley Hamming	RM205
7	12 M	(1914) Hanna Caemmerer Neumann (1921) Kathleen Rita McNulty Mauchly Antonelli	
13	T	(1805) Johann Peter Gustav Lejeune Dirichlet	RM145
14	W	(1468) Johann Werner (1849) Hermann Hankel (1877) Edmund Georg Hermann Landau (1896) Edward Artur Milne (1932) Maurice Audin	RM253 RM063 RM194
15	T	(1564) Galileo Galilei (1850) Sophie Willock Bryant (1861) Alfred North Whitehead (1946) Douglas Hofstadter	RM085
16	F	(1822) Francis Galton (1903) Beniamino Segre	
17	S	(1890) Sir Ronald Aylmer Fisher (1891) Adolf Abraham Halevi Fraenkel (1905) Rózsa Péter	
18	S	(1404) Leon Battista Alberti (1919) Clifford Ambrose Truesdell III	RM157
8	19 M	(1473) Nicolaus Copernicus	RM181
20	T	(1844) Ludwig Boltzmann	RM061
21	W	(1591) Girard Desargues (1915) Evgeny Michailovich Lifshitz	
22	T	(1857) Heinrich Rudolf Hertz (1903) Frank Plumpton Ramsey	RM217
23	F	(1561) Henry Briggs (1583) Jean-Baptiste Morin (1730) Giulio Giuseppe Mozzi del Garbo (1905) Derrick Henry Lehmer (1922) Anneli Cahn Lax (1951) Shigefumi Mori	RM169 RM277 RM215 RM289
24	S	(1871) Felix Bernstein	
25	S	(1827) Henry Watson	
9	26 M	(1786) Dominique Francois Jean Arago	RM193
27	T	(1881) Luitzen Egbertus Jan Brouwer	
28	W	(1735) Alexandre Théophile Vandermonde	RM265
29	T	(1860) Herman Hollerith	RM109

Putnam 2009, A2

Functions f, g, h are differentiable on some open interval around 0 and satisfy the equations and initial conditions

$$f' = 2f^2gh + \frac{1}{gh}, \quad f(0) = 1,$$

$$g' = fg^2h + \frac{4}{fh}, \quad g(0) = 1,$$

$$h' = 3fgh^2 + \frac{1}{fg}, \quad h(0) = 1.$$

Find an explicit formula for $f(x)$, valid in some open interval around 0.

How to put an elephant into a refrigerator

Number Theory

Method 1: Factor the elephant, put the factors in the refrigerator, multiply them together inside the refrigerator.

Method 2: Use induction: if the n^{th} piece fit, the $(n+1)^{\text{th}}$ piece will also fit.

Murphy's Laws applied to Math Students

An expression can be made equal to any other expression if you juggle it enough.

Epur si muove (And yet it does move).

[Apocryphal words to himself after making his abjuration of heliocentricity].

Galileo Galilei

A science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life.

Godfried Harold Hardy

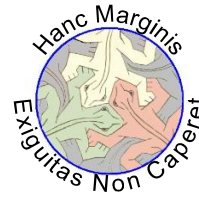
It is well known that the man who first made public the theory of irrationals perished in a shipwreck in order that the inexpressible and unimaginable should ever remain veiled. And so the guilty man, who fortuitously touched on and revealed this aspect of living things, was taken to the place where he began and there is for ever beaten by the waves.

Proclo Diadoco

It is a profoundly erroneous truism, repeated by all copy books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them.

Alfred North Whitehead

1	F	(1611) John Pell (1879) Robert Daniel Carmichael				
	2	S	(1836) Julius Weingarten			
	3	S	(1838) George William Hill (1845) Georg Cantor (1916) Paul Richard Halmos	RM062		
10	4	M	(1822) Jules Antoine Lissajous			
	5	T	(1512) Gerardus Mercator (1759) Benjamin Gompertz (1817) Angelo Genocchi (1885) Pauline Sperry (1915) Laurent Schwartz (1931) Vera Pless	RM230 RM194		
		6	W	(1866) Ettore Bortolotti		
		7	T	(1792) William Herschel (1824) Delfino Codazzi (1922) Olga Alexandrovna Ladyzhenskaya	RM146	
			8	F	(1851) George Chrystal	
		9	S	(1818) Ferdinand Joachimsthal (1900) Howard Hathaway Aiken		
	10	S	(1864) William Fogg Osgood (1872) Mary Ann Elizabeth Stephansen			
		11	11	M	(1811) Urbain Jean Joseph Le Verrier (1853) Salvatore Pincherle (1870) Louis Bachelier	RM158
	12		T	(1685) George Berkeley (1824) Gustav Robert Kirchhoff (1859) Ernesto Cesaro		
			13	W	(1861) Jules Joseph Drach (1957) Rudy D'Alembert	RM278
14	T	(1864) Jozef Kurschak (1879) Albert Einstein (1882) Wacław Sierpiński (1904) Lyudmila Vsevolodovna Keldysh	RM074 RM271 RM266			
	15	F	(1860) Walter Frank Raphael Weldon (1868) Grace Chisolm Young			
16		S	(1750) Caroline Herschel (1789) Georg Simon Ohm (1846) Magnus Gosta Mittag-Leffler	RM146		
	17	S	(1876) Ernest Benjamin Esclangon (1897) Charles Fox (1915) Wolfgang (Vincent) Döblin (Doblin)	RM254		
12		18	M	(1640) Philippe de La Hire (1690) Christian Goldbach (1796) Jacob Steiner (1870) Agnes Sime Baxter	RM122	
		19	T	(1862) Adolf Kneser (1910) Jacob Wolfowitz		
	20		W	(1840) Franz Mertens (1884) Philip Franck (1938) Sergei Petrovich Novikov		
		21	T	(1768) Jean Baptiste Joseph Fourier (1884) George David Birkhoff	RM242	
22	F		(1394) Ulugh Beg (1891) Lorna Mary Swain (1917) Irving Kaplansky (1944) Margaret Hilary Ashworth Millington	RM206		
	23	S	(1749) Pierre-Simon de Laplace (1754) Georg Freiherr von Vega (1882) Emmy Amalie Noether (1897) John Lighton Synge	RM050		
		24	S	(1809) Joseph Liouville (1948) Sun-Yung (Alice) Chang (1966) Gigliola Staffilani	RM142	
13			25	M	(1538) Christopher Clausius	RM290
			26	T	(1848) Konstantin Andreev (1913) Paul Erdős	RM110
27	W	(1857) Karl Pearson				
28	T	(1928) Alexander Grothendieck	RM086			
29	F	(1825) Francesco Faà Di Bruno (1873) Tullio Levi-Civita (1896) Wilhelm Ackermann	RM170 RM098			
	30	S	(1892) Stefan Banach (1921) Alfréd Rényi	RM134		
31		S	(1596) René Descartes	RM218		



March

Putnam 2009, A3

Let d_n be the determinant of the $n \times n$ matrix whose entries, from left to right and then from top to bottom, are $\cos 1, \cos 2, \dots, \cos n^2$. (For example,

$$d_3 = \begin{vmatrix} \cos 1 & \cos 2 & \cos 3 \\ \cos 4 & \cos 5 & \cos 6 \\ \cos 7 & \cos 8 & \cos 9 \end{vmatrix}$$

The argument of \cos is always in radians, not degrees.)

Evaluate $\lim_{n \rightarrow \infty} d_n$.

How to put an elephant into a refrigerator

Algebra

Method 1: Prove that the elephant parts can be placed in the refrigerator, then prove that the refrigerator is closed with respect to addition.

Method 2: Generalize the refrigerator and define a surjection of the refrigerator on the elephant.

Murphy's Laws applied to Math Students

Proofs don't convince anybody of anything.

For example, when I imagine a triangle, although perhaps there is no such figure anywhere in the world outside of my thoughts, and there never has been, nevertheless there does not cease to be a certain nature, or shape or form of this figure. determinate essence, which is immutable and eternal; neither did I invent it, nor does it depend on my spirit in any way; as is clear from the fact that different properties of this triangle can be demonstrated, namely that its three angles are equal to two nets, that the larger angle is subtended by the larger side, and other similar ones, which now, let whether I like it or not, I recognize with all clarity and evidence that I am in it, although I had not previously thought of it in any way, when I first imagined a triangle; and therefore it cannot be said that I composed and invented them.

René Descartes

God does not care about our mathematical difficulties. He integrates empirically.

Albert Einstein

God is a child; and when he started playing, he cultivated mathematics. He is the most divine of human games.

Vinzenz Erath

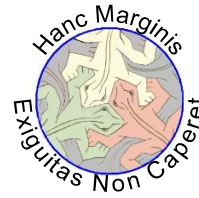
Regular combinations occur more rarely only because they are fewer in number. If we look for a cause where we perceive a symmetry it is not because we consider that a symmetrical event is less possible than the others, but because this event must be the effect of a regular cause or chance, and the first of these suppositions is more probable than the second.

Pierre-Simon De Laplace

When you place a bet at less than the correct odds, which always happens against any organized betting agency, you are paying the operator a percentage fee for the privilege of placing a bet. The probability of winning will be what mathematicians call a "negative expected value." When you follow a system, you make a series of bets, each of which has a negative expected value. There is no way to add minuses to get a plus...

John Scarne

14	1	M	(1640) Georg Mohr (1776) Marie-Sophie Germain (1895) Alexander Craig Aitken	RM219	
	2	T	(1878) Edward Kasner (1934) Paul Joseph Cohen (1984) Alessio Figalli	RM243	
	3	W	(1835) John Howard Van Amringe (1892) Hans Rademacher (1900) Albert Edward Ingham (1971) Alice Riddle		
	4	T	(1809) Benjamin Peirce (1842) François Édouard Anatole Lucas (1949) Shing-Tung Yau	RM123 RM279	
	5	F	(1588) Thomas Hobbes (1607) Honoré Fabri (1622) Vincenzo Viviani (1869) Sergei Alexeievich Chaplygin		
	6	S	(1801) William Hallowes Miller		
	7	S	(1768) François-Joseph Français		
15	8	M	(1903) Marshall Harvey Stone		
	9	T	(1791) George Peacock (1816) Charles Eugene Delaunay (1865) Karl August Rudolph Steinmetz (1894) Cypra Cecilia Krieger Dunaj (1919) John Presper Heckert	RM291	
	10	W	(1857) Henry Ernest Dudeney	RM183	
	11	T	(1953) Andrew John Wiles	RM207	
	12	F	(1794) Germinal Pierre Dandelin (1852) Carl Louis Ferdinand von Lindemann (1903) Jan Tinbergen	RM267	
	13	S	(1728) Paolo Frisi (1813) Duncan Farquharson Gregory (1869) Ada Isabel Maddison (1879) Francesco Severi (1909) Stanislaw Marcin Ulam	RM171	
	14	S	(1629) Christiaan Huygens	RM135	
16	15	M	(1452) Leonardo da Vinci (1548) Pietro Antonio Cataldi (1707) Leonhard Euler (1809) Herman Gunther Grassmann	RM051	
	16	T	(1682) John Hadley (1823) Ferdinand Gotthold Max Eisenstein		
	17	W	(1798) Étienne Bobillier (1853) Arthur Moritz Schonflies (1863) Augustus Edward Hough Love		
	18	T	(1791) Ottaviano Fabrizio Mossotti (1907) Lars Valerian Ahlfors (1918) Hsien Chung Wang (1949) Charles Louis Fefferman	RM150	
	19	F	(1880) Evgeny Evgenievich Slutsky (1883) Richard von Mises (1901) Kiyoshi Oka (1905) Charles Ehresmann		
	20	S	(1839) Francesco Siacchi		
	21	S	(1652) Michel Rolle (1774) Jean Baptiste Biot (1875) Teiji Takagi	RM231	
	17	22	M	(1811) Otto Ludwig Hesse (1887) Harald August Bohr (1935) Bhamu Srinivasan (1939) Sir Michael Francis Atiyah	RM063
		23	T	(1858) Max Karl Ernst Ludwig Planck (1910) Sheila Scott Macintyre	
		24	W	(1863) Giovanni Vailati (1899) Oscar Zariski	RM099
		25	T	(1849) Felix Christian Klein (1900) Wolfgang Pauli (1903) Andrei Nicolayevich Kolmogorov	RM255 RM159
26		F	(1889) Ludwig Josef Johan Wittgenstein		
27		S	(1755) Marc-Antoine Parseval des Chenes (1932) Gian-Carlo Rota	RM195	
28		S	(1906) Kurt Gödel	RM087	
18	29	M	(1854) Jules Henri Poincaré	RM075	
	30	T	(1777) Johann Carl Friedrich Gauss (1916) Claude Elwood Shannon	RM147 RM111	



Putnam 2009, A4

Let S be a set of rational numbers such that

- (a) $0 \in S$;
- (b) If $x \in S$ then $x + 1 \in S$ and $x - 1 \in S$; and
- (c) If $x \in S$ and $x \notin \{0, 1\}$, then $1/(x(x - 1)) \in S$.

Must S contain all rational numbers?

How to put an elephant into a refrigerator

Topology

Method 1: Given the elephant outside the refrigerator, reverse the refrigerator topologically.

Method 2: Use a Klein bottle as a refrigerator.

Method 3: The elephant is compact, so it can be placed in a finite set of refrigerators.

Method 4: Prove that the property of being in the refrigerator is hereditary, then put the elephant's mother in the refrigerator.

Murphy's Laws applied to Math Students

Notes you understood perfectly in class transform themselves into hieroglyphics at home.

In short, Gödel's theorem establishes the impossibility of guaranteeing the non-contradiction of mathematics while remaining within mathematics itself. It seems like a paradox, but the strength of mathematics which should have consisted in its ability to prove every statement logically, now comes to simply demonstrate its own inability to prove. An atmosphere of tragedy, with Gödel in the role of Euripides.

Vittorino Andreoli

In pure mathematics we contemplate absolute truths which existed in the divine mind before the morning stars sang together, and which will continue to exist there when the last of their rays has fallen from heaven.

Edward Everett

The real traditional donut has the topology of a sphere. It is a matter of taste to consider it with separate internal and external surfaces. What is important is that the internal space must be filled with good raspberry jam. This is also a matter of taste.

Peter B. Fellgett

Pauca sed matura [His motto:] Few, but ripe.

Johann Carl Friedrich Gauss

The science of mathematics presents the most brilliant example of how pure reason can successfully expand its domain without the aid of experience.

Immanuel Kant

Mathematical discoveries, small or great are never born of spontaneous generation. They always presuppose a soil seeded with preliminary knowledge and well prepared by labour, both conscious and subconscious.

Jules Henri Poincaré

Anyone who knows a mathematical proposition does not necessarily know something.

Ludwig Josef Johan Wittgenstein

1	W	(1825) Johann Jacob Balmer (1908) Morris Kline (1926) Peter Lax (1977) Maryam Mirzakhani	RM122 RM289 RM189	
2	T	(1860) D'Arcy Wentworth Thompson (1905) Kazimierz Zarankiewicz	RM138	
3	F	(1842) Otto Stolz (1860) Vito Volterra (1892) George Paget Thomson	RM136 RM161	
4	S	(1845) William Kingdon Clifford		
5	S	(1833) Lazarus Emmanuel Fuchs (1883) Anna Johnson Pell Wheeler (1889) René Eugène Gateaux (1897) Francesco Giacomo Tricomi (1923) Cathleen Synge Morawetz	RM196 RM256	
19	6	M	(1872) Willem de Sitter (1906) André Weil	RM088
	7	T	(1854) Giuseppe Veronese (1881) Ebenezer Cunningham (1896) Pavel Sergeievich Alexandrov (1926) Alexis Claude Clairaut	RM220
	8	W	(1859) Johan Ludwig William Valdemar Jensen (1905) Winifred Lydia Caunden Sargent	
	9	T	(1746) Gaspard Monge (1876) Gilbert Ames Bliss (1965) Karen Ellen Smith	RM208
	10	F	(1788) Augustin Jean Fresnel (1847) William Karl Joseph Killing (1904) Edward James Mcshane (1958) Piotr Rezierovich Silverbrahms	
	11	S	(1902) Edna Ernestine Kramer Lassar (1918) Richard Phillips Feynman	RM076
	12	S	(1820) Florence Nightingale (1845) Pierre René Jean Baptiste Henry Brocard (1902) Frank Yates	RM104
20	13	M	(1750) Lorenzo Mascheroni (1899) Pelageia Yakovlevna Polubarinova Kochina	
	14	T	(1832) Rudolf Otto Sigismund Lipschitz (1863) John Charles Fields	RM100
	15	W	(1939) Brian Hartley (1964) Sijue Wu	
	16	T	(1718) Maria Gaetana Agnesi (1821) Pafnuti Lvovi Chebyshev (1911) John (Jack) Todd	RM112 RM139
	17	F	(1940) Alan Kay	
	18	S	(1850) Oliver Heaviside (1892) Bertrand Arthur William Russell	RM160 RM052
	19	S	(1865) Flora Philip (1919) Georgii Dimitirievich Suvorov	
21	20	M	(1861) Henry Seely White	
	21	T	(1471) Albrecht Dürer (1792) Gustave Gaspard de Coriolis	RM124
	22	W	(1865) Alfred Cardew Dixon	
	23	T	(1914) Lipa Bers	RM148
	24	F	(1544) William Gilbert	
	25	S	(1838) Karl Mikailovich Peterson (1979) Elena Tosato	RM268
	26	S	(1667) Abraham de Moivre (1896) Yuri Dimitrievich Sokolov	RM280
22	27	M	(1862) John Edward Campbell	
	28	T	(1676) Jacopo Francesco Riccati (1710) Johann (II) Bernoulli	RM232 RM093
	29	W	(1882) Harry Bateman	
	30	T	(1814) Eugene Charles Catalan	RM184
	31	F	(1912) Wu Chien Shiung (Madame Wu) (1926) John Kemeny	RM292



Putnam 2009, A5

Is there a finite abelian group G such that the product of the orders of all its elements is 2^{2009} ?

How to put an elephant into a refrigerator

Algebraic Topology

Replace the inside of the refrigerator with its universal cover (\mathbb{R}^n).

Murphy's Laws applied to Math Students

Textbooks are written for those who already know the subject.

Deduction, which takes us from the general proposition to facts again – teaches us, if I may so say, to anticipate from the ticket what is inside the bundle.

Thomas Henry Huxley

“But,” you might say, “none of this shakes my belief that 2 and 2 are 4.” You are quite right, except in marginal cases – and it is only in marginal cases that you are doubtful whether a certain animal is a dog or a certain length is less than a meter. Two must be two of something, and the proposition “2 and 2 are 4” is useless unless it can be applied. Two dogs and two dogs are certainly four dogs, but cases arise in which you are doubtful whether two of them are dogs. “Well, at any rate there are four animals”, you may say. But there are microorganisms concerning which it is doubtful whether they are animals or plants. “Well, then living organisms”, you say. But there are things of which it is doubtful whether they are living organisms or not. You will be driven into saying: “Two entities and two entities are four entities.” When you have told me what you mean by “entity”, we will resume the argument.

Bertrand Arthur William Russell

Some people think that, within mathematics, arithmetic, algebra and geometry are separate subjects: this is a serious mistake. Instead, they all collaborate with each other, help each other, and are sometimes interchangeable.

Malba Tahan

*When I heard the cultured astronomer,
When demonstrations and figures were lined up before me,
when they showed me maps and diagrams to add, divide and measure them,
when I sat down to listen to the astronomer's seminar amidst a thousand applause in the room,
oh, how soon I got tired and fed up,
until I got up and slipped away, running away,
in the mystical misty night air, and from time to time
I gazed at the stars in perfect silence.*

Walt Whitman

1	S	(1796) Sadi Leonard Nicolas Carnot (1851) Edward Bailey Elliott (1899) Edward Charles Titchmarsh		
2	S	(1895) Tibor Radó		
23	3	M	(1659) David Gregory (1954) Susan Landau	
4	T	(1809) John Henry Pratt (1966) Svetlana Yakovlevna Jitomirskaya	RM197	
5	W	(1814) Pierre Laurent Wantzel (1819) John Couch Adams (1883) John Maynard Keynes (1941) Nikolai Vladimirovic Krylov	RM065 RM281 RM269 RM286	
6	T	(1436) Johann Müller Regiomontanus (1857) Aleksandr Michailovitch Lyapunov (1906) Max August Zorn	RM185 RM077	
7	F	(1863) Edward Burr Van Vleck		
8	S	(1625) Giovanni Domenico Cassini (1858) Charlotte Angas Scott (1860) Alicia Boole Stott (1896) Eleanor Pairman (1923) Gloria Olive (1924) Samuel Karlin	RM245 RM209	
9	S	(1885) John Edensor Littlewood	RM049	
24	10	M	(940) Mohammad Abu'L Wafa Al-Buzjani (1887) Vladimir Ivanovich Smirnov	RM257 RM101
11	T	(1881) Hilda Phoebe Hudson (1937) David Bryant Mumford		
12	W	(1888) Zygmunt Janyszewski (1937) Vladimir Igorevich Arnold	RM221	
13	T	(1831) James Clerk Maxwell (1872) Jessie Chrystal Macmillan (1876) William Sealey Gosset (Student) (1928) John Forbes Nash	RM113 RM149	
14	F	(1736) Charles Augustin de Coulomb (1856) Andrei Andreyevich Markov (1903) Alonzo Church	RM125 RM233	
15	S	(1640) Bernard Lamy (1894) Nikolai Gregorievich Chebotaryov		
16	S	(1915) John Wilder Tukey		
25	17	M	(1898) Maurits Cornelius Escher	RM097
18	T	(1858) Andrew Russell Forsyth (1884) Charles Ernest Weatherburn (1884) Frieda Nudel (1913) Paul Teichmüller (1915) Alice Turner Schafer	RM148	
19	W	(1623) Blaise Pascal (1902) Wallace John Eckert	RM053	
20	T	(1873) Alfred Loewy (1917) Helena Rasiowa		
21	F	(1781) Simeon Denis Poisson (1828) Giuseppe Bruno (1870) Clara Immerwahr	RM293 RM182	
22	S	(1822) Mario Pieri (1864) Hermann Minkowsky (1910) Konrad Zuse (1932) Mary Wynne Warner		
23	S	(1912) Alan Mathison Turing	RM089	
26	24	M	(1880) Oswald Veblen	
25	T	(1908) William Van Orman Quine		
26	W	(1824) William Thomson, Lord Kelvin (1918) Yudell Leo Luke	RM161	
27	T	(1806) Augustus de Morgan		
28	F	(1875) Henri Léon Lebesgue	RM173	
29	S	(1888) Aleksandr Aleksandrovich Friedmann (1979) Artur Avila Cordeiro de Melo	RM101 RM189	
30	S	(1791) Felix Savart (1958) Abigail Thompson		



Putnam 2009, A6

Let $f: [0; 1]^2 \rightarrow \mathbb{R}$ be a continuous function on the closed unit square such that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous on the interior $(0, 1)^2$. Let $a = \int_0^1 f(0, y) dy$, $b = \int_0^1 f(1, y) dy$, $c = \int_0^1 f(x, 0) dx$, $d = \int_0^1 f(x, 1) dx$. Prove or disprove: There must be a point $(x_0; y_0)$ in $(0; 1)^2$ such that

$$\frac{\partial f}{\partial x}(x_0, y_0) = b - a \text{ and } \frac{\partial f}{\partial y}(x_0, y_0) = d - c.$$

How to put an elephant into a refrigerator

Linear Algebra

Method 1: Define an orthonormal basis for the elephant and place the basis in the refrigerator.

Method 2: Show that part of the elephant can fit into the refrigerator; if the operator is linear, all parts can fit into the refrigerator.

Murphy's Laws applied to Math Students

Any simple idea can be expressed in incomprehensible terms.

If you add 3 and 5, you get 8; the three-ity of 3 and the five-ity of 5 have merged and lost in the eight-ity of 8, like two drops of water coming together. However, if you add 3 to 5i, you get the complex number 3+5i, a drop of water and a drop of oil – linear independence.

John Derbyshire

Mathematics seems ever to teach us two lessons: there is no limit to our mind's ingenuity; and there is even less of a limit to the intransigence of the world.

Robert & Ellen Kaplan

Unfortunately, as soon as people talk about huge numbers, they seem to go crazy. They seem to be under the impression that since zero equals nothing, they can add any number of zeros to a number without any practical consequences.

E. Kasner, J.R. Newman

A precision professor had the habit of saying: "... quartic polynomial $ax^4 + bx^3 + cx^2 + dx + e$, where e need not be the base of the natural logarithms."

John Edensor Littlewood

Words differently arranged have a different meaning and meanings differently arranged have a different effect.

Blaise Pascal

Sometimes a proof introduces the mathematicians into a whole new world of mathematical ideas that would never have been known without that proof.

Igor Rostislavovich Shafarevich



27	1	M	(1643) Gottfried Wilhelm von Leibniz (1788) Jean-Victor Poncelet (1906) Jean Alexandre Eugène Dieudonné	RM054 RM246	
	2	T	(1820) William John Rankine (1852) William Burnside (1925) Olga Arsen'evna Oleinik		
	3	W	(1807) Ernest Jean Philippe Fauque de Jonquières (1897) Jesse Douglas	RM162	
	4	T	(1906) Daniel Edwin Rutherford (1917) Michail Samoilovich Livsic		
	5	F	(1936) James Mirrlees		
	6	S	(1849) Alfred Bray Kempe		
	7	S	(1816) Johann Rudolf Wolf (1906) William Feller (1922) Vladimir Aleksandrovich Marchenko		
28	8	M	(1760) Christian Kramp (1904) Henri Paul Cartan	RM126	
	9	T	(1845) George Howard Darwin (1931) Valentina Mikhailovna Borok	RM138 RM197	
	10	W	(1856) Nikola Tesla (1862) Roger Cotes (1868) Oliver Dimon Kellogg	RM174	
	11	T	(1857) Sir Joseph Larmor (1888) Jacob David Tamarkin (1890) Giacomo Albanese	RM101	
	12	F	(1875) Ernest Sigismund Fischer (1895) Richard Buckminster Fuller (1935) Nicolas Bourbaki	RM066 RM126	
	13	S	(1527) John Dee (1741) Karl Friedrich Hindenburg	RM234	
	14	S	(1671) Jacques D'Allonville (1793) George Green	RM078	
	29	15	M	(1865) Wilhelm Wirtinger (1898) Mary Taylor Slow (1906) Adolph Andrej Pavlovich Yushkevich	
		16	T	(1678) Jakob Hermann (1903) Irmgard Flugge-Lotz	
		17	W	(1831) Victor Mayer Amédeé Mannheim (1837) Wilhelm Lexis (1944) Krystyna Maria Trybulec Kuperberg	
		18	T	(1013) Hermann von Reichenau (1635) Robert Hooke (1853) Hendrik Antoon Lorentz	RM282 RM114 RM161
		19	F	(1768) Francois Joseph Servois	
		20	S	(1876) Otto Blumenthal (1947) Gerd Binnig	RM258 RM222
		21	S	(1620) Jean Picard (1848) Emil Weyr (1849) Robert Simpson Woodward (1861) Herbert Ellsworth Slaught	
30		22	M	(1784) Friedrich Wilhelm Bessel	RM198
		23	T	(1775) Étienne-Louis Malus (1854) Ivan Slezynsky	
		24	W	(1851) Friedrich Hermann Schottky (1871) Paul Epstein (1923) Christine Mary Hamill	
		25	T	(1808) Johann Benedict Listing	
	26	F	(1903) Kurt Mahler		
	27	S	(1667) Johann Bernoulli (1801) George Biddel Airy (1848) Lorand Baron von Eötvös (1867) Derrick Norman Lehmer (1871) Ernst Friedrich Ferdinand Zermelo	RM093 RM210 RM215 RM090	
	28	S	(1954) Gerd Faltings	RM222	
	31	29	M	(1898) Isidor Isaac Rabi	RM294
		30	T	(1889) Vladimir Kosma Zworoklyn	
		31	W	(1704) Gabriel Cramer (1712) Johann Samuel Koenig (1926) Hilary Putnam	RM186

Putnam 2009, B1

Show that every positive rational number can be written as a quotient of products of factorials of (not necessarily distinct) primes. For example,

$$\frac{10}{9} = \frac{2! \cdot 5!}{3! \cdot 3! \cdot 3!}$$

How to put an elephant into a refrigerator

Affine geometry

There is a similar transformation that puts the elephant into the refrigerator.

Murphy's Laws applied to Math Students

The answers you need are not at the back of the book.

An anagram, an acrostic (only in part), a palindrome, a crossword puzzle function like so many mathematical equations: the solver who faces them knows that in his resolution he will be helped by the necessity that every element present on one side of the equal sign must also be present on the other, and that given the unknowns the final account must be zero-sum. In these cases, puzzles are a language subjected to the litmus test.

Claudio Bartocci

There will come a time when the proper education of children, by a glorified system of spontaneous education of choice, similar to the Montessori System, will be made possible. Children, as well as grown-ups, in their individual, glorified, drudgery-proof homes of Labrador, the tropics, the Orient, or where you will, to which they can pass with pleasure and expedition by means of ever-improving transportation, will be able to tune in their television and radio to the moving picture lecture of, let us say, President Lowell of Harvard; the professor of Mathematics of Oxford; of the doctor of Indian antiquities of Delhi, etc. Education by choice, with its marvelous motivating psychology of desire for truth, will make life ever cleaner and happier, more rhythmical and artistic.

Richard Buckminster Fuller

By the help of microscopes, there is nothing so small, as to escape our inquiry; hence there is a new visible world discovered to the understanding.

Robert Hooke

A person's aesthetic and intellectual life cannot be complete unless it includes an appreciation for the power and beauty of mathematics. To put it simply, aesthetic and intellectual fulfilment requires knowing a little math.

Jerry P. King

I do not think there is any thrill that can go through the human heart like that felt by the inventor as he sees some creation of the brain unfolding to success... Such emotions make a man forget food, sleep, friends, love, everything.

Nikola Tesla

	1	T	(1861) Ivar Otto Bendixson (1881) Otto Toeplitz (1955) Bernadette Perrin-Riou	
	2	F	(1856) Ferdinand Rudio (1902) Mina Spiegel Rees	
	3	S	(1914) Mark Kac	RM115
	4	S	(1805) Sir William Rowan Hamilton (1838) John Venn	RM079
32	5	M	(1802) Niels Henrik Abel (1941) Alexander Keewatin Dewdney	RM055
	6	T	(1638) Nicolas Malebranche (1741) John Wilson	RM283
	7	W	(1868) Ladislaus Josephowitsch Bortkiewitz	
	8	T	(1902) Paul Adrien Maurice Dirac (1931) Sir Roger Penrose (1974) Manjul Bhargava	RM103 RM189
	9	F	(1537) Francesco Barozzi (Franciscus Barocius) (1940) Linda Goldway Keen	RM223
	10	S	(1602) Gilles Personne de Roberval (1901) Franco Dino Rasetti (1917) Nikolai Sergeevitc Krylov (1926) Carol Ruth Karp	RM235 RM286
	11	S	(1730) Charles Bossut (1842) Enrico D'Ovidio	RM259
33	12	M	(1882) Jules Antoine Richard (1887) Erwin Rudolf Josef Alexander Schrödinger	RM103
	13	T	(1625) Erasmus Bartholin (1819) George Gabriel Stokes (1861) Cesare Burali-Forti	RM187
	14	W	(1530) Giovanni Battista Benedetti (1842) Jean Gaston Darboux (1865) Guido Castelnuovo (1866) Charles Gustave Nicolas de La Vallée-Poussin	
	15	T	(1863) Aleksei Nikolaevich Krylov (1892) Louis Pierre Victor Duc de Broglie (1901) Piotr Sergeevich Novikov	RM286 RM175
	16	F	(1773) Louis-Benjamin Francoeur (1821) Arthur Cayley	
	17	S	(1601) Pierre de Fermat	RM091
	18	S	(1685) Brook Taylor	
34	19	M	(1646) John Flamsteed (1739) Georg Simon Klügel	
	20	T	(1710) Thomas Simpson (1863) Corrado Segre	RM247
	21	W	(1789) Augustin-Louis Cauchy	RM127
	22	T	(1647) Denis Papin	
	23	F	(1683) Giovanni Poleni (1829) Moritz Benedikt Cantor (1842) Osborne Reynolds	
	24	S	(1561) Bartholomeo Pitiscus (1942) Karen Keskulla Uhlenbeck	RM163
	25	S	(1561) Philip Van Lansberge (1844) Thomas Muir	RM199
35	26	M	(1728) Johann Heinrich Lambert (1875) Giuseppe Vitali (1965) Marcus Peter Francis du Sautoy	
	27	T	(1858) Giuseppe Peano	RM067
	28	W	(1796) Irénée Jules Bienaymé (1862) Roberto Marcolongo	RM187
	29	T	(1904) Leonard Roth	
	30	F	(1703) Giovanni Ludovico Calandrini (1856) Carle David Tolmé Runge (1906) Olga Taussky-Todd	RM186 RM139
	31	S	(1821) Hermann Ludwig Ferdinand von Helmholtz (1885) Herbert Westren Turnbull	RM211



Putnam 2009, B2

A game involves jumping to the right on the real number line. If a and b are real numbers and $b > a$, the cost of jumping from a to b is $b^3 - ab^2$. For what real numbers c can one travel from 0 to 1 in a finite number of jumps with total cost exactly c ?

How to put an elephant into a refrigerator

Set Theory

Since $\{\text{Refrigerator}\} = \{\text{Elephant}\}$, the elephant and the refrigerator have the same cardinality.

Murphy's Laws applied to Math Students

No matter how much you studied for any exams, it will never be enough.

I shall devote all my efforts to bring light into the immense obscurity that today reigns in Analysis. It so lacks any plan or system, that one is really astonished that there are so many people who devote themselves to it – and, still worse, it is absolutely devoid of any rigor.

Niels Henrik Abel

...the certitude of mathematics is contained in the syntactic rigour of demonstrations.

Francesco Barozzi

This is the main fault of the doctrinaire spirit that invades our school. We teach to be wary of approximation, which is reality, to adopt the idol of a perfection which is illusory.

[...] if for the sake of culture, we suffocate the practical sense and spirit of initiative in these disciples, we are failing in the greatest of our duties.

Guido Castelnuovo

God is a mathematician of a very high order and He used advanced mathematics in constructing the universe.

Paul Adrien Maurice Dirac

Mathematics is the most tolerant of disciplines, it is based on a single certainty: if two people discuss for long enough in the end they must agree, perhaps on which hypotheses to assume, and they will find an agreement that they will both be ready to defend with the same determination towards others.

Furio Honsell

Games are among the most interesting creations of the human mind, and the analysis of their structure is full of adventure and surprises. Unfortunately, there is never a lack of mathematicians for the job of transforming delectable ingredients into a dish that tastes like a damp blanket.

James R. Newman

	1	S	(1647) Giovanni Ceva (1659) Joseph Saurin (1835) William Stanley Jevons	RM203	
36	2	M	(1878) Maurice René Frechet (1923) René Thom	RM080	
	3	T	(1814) James Joseph Sylvester (1884) Solomon Lefschetz (1908) Lev Semenovich Pontryagin	RM104	
	4	W	(1809) Luigi Federico Menabrea	RM150	
	5	T	(1667) Giovanni Girolamo Saccheri (1725) Jean-Étienne Montucla	RM128	
	6	F	(1859) Boris Jakovlevich Bukreev (1863) Dimitri Aleksandrovich Grave		
	7	S	(1707) George Louis Leclerc Comte de Buffon (1948) Cheryl Elisabeth Praeger (1955) Efim Zelmanov		
	8	S	(1584) Gregorius Saint-Vincent (1588) Marin Mersenne	RM092	
	37	9	M	(1860) Frank Morley (1914) Marjorie Lee Browne	
10		T	(1839) Charles Sanders Peirce	RM123	
11		W	(1623) Stefano degli Angeli (1798) Franz Ernst Neumann (1877) Sir James Hopwood Jeans	RM224	
12		T	(1891) Antoine André Louis Reynaud (1894) Dorothy Maud Wrinch (1900) Haskell Brooks Curry	RM260 RM212	
13		F	(1873) Constantin Carathéodory (1885) Wilhelm Johann Eugen Blaschke		
14		S	(1858) Henry Burchard Fine (1891) Ivan Matveevich Vinogradov		
15		S	(973) Abu Arrayhan Muhammad Ibn Ahmad Al'Biruni (1886) Paul Pierre Levy	RM164	
38	16	M	(1494) Francisco Maurolico (1736) Johann Nikolaus Tetens	RM296	
	17	T	(1743) Marie Jean Antoine Nicolas de Caritat de Condorcet (1826) Georg Friedrich Bernhard Riemann	RM176 RM068	
	18	W	(1752) Adrien-Marie Legendre	RM140	
	19	T	(1749) Jean-Baptiste Delambre		
	20	F	(1842) Alexander Wilhelm von Brill (1861) Frank Nelson Cole		
	21	S	(1899) Juliusz Pawel Schauder (1917) Phyllis Nicolson		
	22	S	(1765) Paolo Ruffini (1769) Louis Puissant (1803) Jaques Charles Francois Sturm	RM116	
	39	23	M	(1768) William Wallace (1900) David Van Dantzig	
		24	T	(1501) Girolamo Cardano (1625) Johan de Witt (1801) Michail Vasilevich Ostrogradski (1862) Winifred Edgerton Merrill (1945) Ian Nicholas Stewart	RM064 RM188 RM056 RM236
		25	W	(1819) George Salmon (1888) Stefan Mazurkiewicz	
26		T	(1688) Willem Jakob 's Gravesande (1854) Percy Alexander Macmahon (1891) Hans Reichenbach		
27		F	(1855) Paul Émile Appell (1876) Earle Raymond Hedrick (1919) James Hardy Wilkinson		
28		S	(1698) Pierre Louis Moreau de Maupertuis (1761) Ferdinand François Desiré Budan de Boislaurent (1873) Julian Lowell Coolidge	RM152	
29		S	(1540) François Viète (1561) Adriaan Van Roomen (1812) Adolph Gopel	RM200 RM200	
40		30	M	(1775) Robert Adrain (1829) Joseph Wolstenholme (1883) Ernst Hellinger (1891) Otto Yuljevich Schmidt	RM248



Putnam 2009, B3

Call a subset S of $\{1, 2, \dots, n\}$ *mediocre* if it has the following property: Whenever a and b are elements of S whose average is an integer, that average is also an element of S . Let $A(n)$ be the number of mediocre subsets of $\{1, 2, \dots, n\}$. [For instance, every subset of $\{1, 2, 3\}$ except $\{1, 3\}$ is mediocre, so $A(3) = 7$.] Find all positive integers n such that $A(n+2) - 2A(n+1) + A(n) = 1$.

How to put an elephant into a refrigerator

Geometry

Axiom: An elephant can be put in the refrigerator.

Murphy's Laws applied to Math Students

The problems you know how to solve never come up in the exam.

There is a difficulty connected to infinity that concerns the mathematician. If infinity is not actual and the size of the universe is finite, his theorems on numbers will not be true for all numbers, but only for a finite number of them; and the mathematician will not be able to extend his lines and planes indefinitely to demonstrate certain geometric theorems.

Aristotle

Shouting at the monkeys in the tree opposite. That's what brains evolved into. Not math or physics.

Jack Cohen, Terry Pratchett, Ian Stewart

Enraged, the Beast twisted its double and triple integrals to counter the polynomials with which the king tried to hit it, slipped into an infinite series of indeterminate terms, then rose again, raising itself to a power, but the king attacked it with a series of partial derivations and totals to zero all its coefficients (see Riemann's Lemma), and in the ensuing confusion the builders completely lost sight of the king and the beast.

Stanislaw Lem

Despite all the experience I may have gained in music from having associated myself so much with it, I must confess that only with the help of mathematics have my ideas become clear.

Jean-Philippe Rameau

No discipline is more suited than mathematics to giving the sense, to those who possess it, of an indestructible spiritual treasure, a set of solid knowledge that can never prove to be incorrect.

Gaetano Scorza

Since Hardy's time the world has also changed. A typical day for the great Cambridge scholar consisted of a maximum of four hours of intense reflection on research problems while the rest of the time was spent playing cricket, Hardy's great passion in addition to mathematics, and reading newspapers. There probably remained space for some sporadic meetings with students, but Hardy was reticent on personal matters.

Ian Nicholas Stewart

1	T	(1671) Luigi Guido Grandi (1898) Bela Kerekjarto' (1912) Kathleen Timpson Ollerenshaw	RM177
2	W	(1825) John James Walker (1908) Arthur Erdélyi	
3	T	(1944) Pierre René Deligne	
4	F	(1759) Louis Francois Antoine Arbogast (1797) Jerome Savary	
5	S	(1732) Nevil Maskelyne (1781) Bernhard Placidus Johann Nepomuk Bolzano (1861) Thomas Little Heath	RM117
6	S	(1552) Matteo Ricci (1831) Julius Wilhelm Richard Dedekind (1908) Sergei Lvovich Sobolev	RM141 RM081
41	7	M	(1885) Niels Bohr RM063
	8	T	(1908) Hans Arnold Heilbronn
	9	W	(1581) Claude Gaspard Bachet de Meziriac (1704) Johann Andrea von Segner (1854) Mihajlo Idvorski Pupin (1873) Karl Schwarzschild (1949) Fan Rong K Chung Graham RM201 RM297 RM153 RM110
	10	T	(1731) Henry Cavendish (1861) Heinrich Friedrich Karl Ludwig Burkhardt RM273
	11	F	(1675) Samuel Clarke (1777) Barnabè Brisson (1881) Lewis Fry Richardson (1885) Alfred Haar (1910) Cahit Arf RM261
	12	S	(1860) Elmer Sperry
	13	S	(1890) Georg Feigl (1893) Kurt Werner Friedrich Reidemeister (1932) John Griggs Thomson
42	14	M	(1687) Robert Simson (1801) Joseph Antoine Ferdinand Plateau (1868) Alessandro Padoa
	15	T	(1608) Evangelista Torricelli (1735) Jesse Ramsden (1776) Peter Barlow (1931) Eléna Wexler-Kreindler RM165
	16	W	(1879) Philip Edward Bertrand Jourdain
	17	T	(1759) Jacob (II) Bernoulli (1888) Paul Isaac Bernays RM093
	18	F	(1945) Margaret Dusa Waddington Mcduff RM249
	19	S	(1903) Jean Frédéric Auguste Delsarte (1910) Subrahmanyam Chandrasekhar RM153
	20	S	(1632) Sir Christopher Wren (1863) William Henry Young (1865) Aleksandr Petrovich Kotelnikov RM105
43	21	M	(1677) Nicolaus (I) Bernoulli (1823) Enrico Betti (1855) Giovan Battista Guccia (1893) William Leonard Ferrar (1914) Martin Gardner RM093 RM150 RM129 RM137
	22	T	(1587) Joachim Jungius (1895) Rolf Herman Nevanlinna (1907) Sarvadaman Chowla RM285
	23	W	(1865) Piers Bohl
	24	T	(1804) Wilhelm Eduard Weber (1873) Edmund Taylor Whittaker
	25	F	(1811) Évariste Galois RM069
	26	S	(1849) Ferdinand Georg Frobenius (1857) Charles Max Mason (1911) Shiing-Shen Chern
	27	S	(1678) Pierre Remond de Montmort (1856) Ernest William Hobson
44	28	M	(1804) Pierre François Verhulst
	29	T	(1925) Klaus Roth
	30	W	(1906) Andrej Nikolaevich Tichonov (1946) William Paul Thurston RM237
	31	T	(1711) Laura Maria Caterina Bassi (1815) Karl Theodor Wilhelm Weierstrass (1935) Ronald Lewis Graham RM189 RM057 RM110



Putnam 2009, B4

Say that a polynomial with real coefficients in two variables, x, y is *balanced* if the average value of the polynomial on each circle centered at the origin is 0. The balanced polynomials of degree at most 2009 form a vector space V over \mathbb{R} . Find the dimension of V .

How to put an elephant into a refrigerator

Complex Analysis

Put the refrigerator at the origin and the elephant outside the unit circle, then you get the image by inversion.

Murphy's Laws applied to Math Students

The problem that definitely won't come up on the exam, will come up on the exam.

One may ask the question as to the extent to which the quest for beauty is an aim in the pursuit of science.... It is, indeed, an incredible fact that what the human mind, at its deepest and most profound, perceives as beautiful finds its realization in external nature. What is intelligible is also beautiful.

Subrahmanyam Chandrasekhar

[From the preface to his final manuscript.] Since the beginning of the century, computational procedures have become so complicated that any progress by those means has become impossible, without the elegance which modern mathematicians have brought to bear on their research, and by means of which the spirit comprehends quickly and in one step a great many computations.

It is clear that elegance, so vaunted and so aptly named, can have no other purpose. ...

Go to the roots, of these calculations! Group the operations. Classify them according to their complexities rather than their appearances! This, I believe, is the mission of future mathematicians. This is the road on which I am embarking in this work.

Évariste Galois

Well, as you know, there are 24 hours in every day. And if that's not enough, you've always got the nights!

Ronald Lewis Graham

What is brought to light by the intellect cannot condescend to falsehood.

Matteo Ricci

Geometry alone, among the liberal disciplines, exercises and sharpens the ingenuity and makes it suitable to be an ornament of the city in peace and to defend it in war [...] other things being equal, in fact, the ingenuity that is exercised in geometric gymnastics he possesses a very particular and virile strength.

Evangelista Torricelli

	1	F	(1535) Giambattista della Porta	RM226
	2	S	(1815) George Boole (1826) Henry John Stephen Smith	RM094
	3	S	(1867) Martin Wilhelm Kutta (1878) Arthur Byron Coble (1896) Raymond Louis Wilder (1906) Carl Benjamin Boyer	
45	4	M	(1744) Johann (III) Bernoulli (1865) Pierre Simon Girard	RM093
	5	T	(1848) James Whitbread Lee Glaisher (1930) John Frank Adams	
	6	W	(1906) Emma Markovna Trotskaia Lehmer	RM215
	7	T	(1660) Thomas Fantet de Lagny (1799) Karl Heinrich Graffe (1867) Maria Skłodowska Curie (1878) Lise Meitner (1898) Raphael Salem	RM182 RM238
	8	F	(1656) Edmond Halley (1781) Giovanni Antonio Amedeo Plana (1846) Eugenio Bertini (1848) Friedrich Ludwig Gottlob Frege (1854) Johannes Robert Rydberg (1869) Felix Hausdorff	RM190 RM154 RM274 RM178
	9	S	(1847) Carlo Alberto Castigliano (1885) Theodor Franz Eduard Kaluza (1885) Hermann Klaus Hugo Weyl (1906) Jaroslav Borisovich Lopatynsky (1913) Hedwig Eva Maria Kiesler (Hedy Lamarr) (1922) Imre Lakatos	RM202 RM082 RM144
	10	S	(1829) Helwin Bruno Christoffel	
46	11	M	(1904) John Henry Constantine Whitehead	
	12	T	(1825) Michail Egorovich Vashchenko-Zakharchenko (1842) John William Strutt Lord Rayleigh (1927) Yutaka Taniyama	
	13	W	(1876) Ernest Julius Wilkzynsky (1878) Max Wilhelm Dehn	
	14	T	(1845) Ulisse Dini (1919) Paulette Libermann (1975) Martin Hairer	RM189
	15	F	(1688) Louis Bertrand Castel (1793) Michel Chasles (1794) Franz Adolph Taurinus	
	16	S	(1835) Eugenio Beltrami	RM262
	17	S	(1597) Henry Gellibrand (1717) Jean-Baptiste Le Rond D'Alembert (1790) August Ferdinand Möbius (1902) Eugene Wigner	RM166 RM118 RM298
47	18	M	(1872) Giovanni Enrico Eugenio Vacca (1927) Jon Leslie Britton	
	19	T	(1894) Heinz Hopf (1900) Michail Alekseevich Lavrentev (1901) Nina Karlovna Bari	RM214
	20	W	(1889) Edwin Powell Hubble (1924) Benoît Mandelbrot (1963) William Timothy Gowers	
	21	T	(1867) Dimitri Sintsov	
	22	F	(1803) Giusto Bellavitis (1840) Émile Michel Hyacinthe Lemoine	
	23	S	(1616) John Wallis (1820) Issac Todhunter (1917) Elizabeth Leonard Scott	RM070 RM106
	24	S	(1549) Duncan Maclaren Young Sommerville (1909) Gerhard Gentzen	
48	25	M	(1841) Fredrich Wilhelm Karl Ernst Schröder (1873) Claude Louis Mathieu (1943) Evelyn Merle Roden Nelson	
	26	T	(1894) Norbert Wiener (1946) Enrico Bombieri	RM172
	27	W	(1867) Arthur Lee Dixon	
	28	T	(1898) John Wishart	
	29	F	(1803) Christian Andreas Doppler (1849) Sir Horace Lamb (1879) Nikolay Mitrofanovich Krylov	RM250 RM286
	30	S	(1549) Sir Henry Savile (1969) Matilde Marcolli	RM142



Putnam 2009, B5

Let $f: (1, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that

$$f'(x) = \frac{x^2 - (f(x))^2}{x^2((f(x))^2 + 1)} \text{ for all } x > 1.$$

Prove that $\lim_{x \rightarrow \infty} f(x) = \infty$.

How to put an elephant into a refrigerator

Numerical Analysis

Place the elephant's trunk in the refrigerator and define the rest of the elephant as a rounding error.

Murphy's Laws applied to Math Students

The answer to the problem you couldn't solve during the exam will become obvious after you hand in your paper.

The fact that its arguments can in principle be resolved makes mathematics unique. There is no mathematical equivalent of astronomers who still believe in the steady-state cosmological theory, or of biologists who with great conviction hold very different ideas about what can be explained by natural selection, or of philosophers who fundamentally disagree about the relationship between awareness and the physical world, or of economists who follow different schools of thought such as monetarism and neo-Keynesianism.

William Timothy Gowers

Every spiritual audacity today rests on the exact sciences. We do not learn from Goethe, Hebbel, Hölderlin, but from Mach, Lorentz, Einstein, Minkowski, from Couturat, Russell, Peano [...] The program of every single work of art can be this: mathematical audacity, dissolution of consciousness in the elements, unlimited permutation of these elements; everything is related to everything else, and develops from this.

Robert Musil

Man is confined within the narrow limits of the body, as in a prison, but mathematics frees him, and makes him greater than the entire universe. [...] Tossed here and there, aimlessly, by the storm of passions, mathematics restores his inner peace, harmoniously resolving the opposing movements of the soul, and bringing it back, under the guidance of reason, to agreement and harmony.

Petrus Ramus

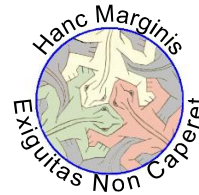
The superstitious masses hold it as an axiom that God's judgments far transcend human intellect. Such a doctrine could be enough to hide the truth from the human race for all eternity, if mathematics had not provided us with another comparison of truth, considering only the essences and properties of things without taking into account their final causes.

Baruch Spinoza

Anyone who has heard the same thing said by 12,000 eyewitnesses has only 12,000 probabilities, which are equal to a strong probability, which in turn is far from certainty.

Voltaire

	1	S	(1792) Nikolay Yvanovich Lobachevsky (1847) Christine Ladd-Franklin	RM083	
49	2	M	(1831) Paul David Gustav du Bois-Reymond (1901) George Frederick James Temple		
	3	T	(1903) Sidney Goldstein (1924) John Backus		
	4	W	(1795) Thomas Carlyle		
	5	T	(1868) Arnold Johannes Wilhelm Sommerfeld (1901) Werner Karl Heisenberg (1907) Giuseppe Occhialini	RM275 RM155 RM122	
	6	F	(1682) Giulio Carlo Fagnano dei Toschi		
	7	S	(1823) Leopold Kronecker (1830) Antonio Luigi Gaudenzio Giuseppe Cremona (1924) Mary Ellen Rudin	RM239 RM150	
	8	S	(1508) Regnier Gemma Frisius (1865) Jaques Salomon Hadamard (1919) Julia Bowman Robinson	RM263 RM227	
	50	9	M	(1883) Nikolai Nikolaievich Luzin (1906) Grace Brewster Murray Hopper (1917) Sergei Vasilovich Fomin	RM214
		10	T	(1804) Karl Gustav Jacob Jacobi (1815) Augusta Ada King Countess Of Lovelace	RM251 RM059
		11	W	(1882) Max Born	RM155
		12	T	(1832) Peter Ludwig Mejdell Sylow (1913) Emma Castelnuovo	RM191
		13	F	(1724) Franz Ulrich Theodosius Aepinus (1887) George Pólya	RM131
		14	S	(1546) Tycho Brahe	
		15	S	(1802) János Bolyai (1923) Freeman John Dyson	RM083
		51	16	M	(1804) Wiktor Yakovievich Bunyakowsky
17	T		(1706) Gabrielle Émilie Le Tonnelier de Breteuil du Châtelet (1835) Felice Casorati (1842) Marius Sophus Lie (1900) Dame Mary Lucy Cartwright	RM299	
18	W		(1856) Joseph John Thomson (1917) Roger Lyndon (1942) Lenore Blum	RM161	
19	T		(1783) Charles Julien Brianchon (1854) Marcel Louis Brillouin (1887) Charles Galton Darwin	RM138	
20	F		(1494) Oronce Fine (1648) Tommaso Ceva (1737) Tommaso Valperga di Caluso (1875) Francesco Paolo Cantelli	RM203 RM287	
21	S		(1878) Jan Łukasiewicz (1921) Edith Hirsch Luchins (1932) John Robert Ringrose		
22	S		(1824) Francesco Brioschi (1859) Otto Ludwig Hölder (1869) Dimitri Fedorovich Egorov (1877) Tommaso Boggio (1887) Srinivasa Aiyangar Ramanujan	RM150 RM214	
52	23		M	(1872) Georgii Yurii Pfeiffer	
	24		T	(1822) Charles Hermite (1868) Emmanuel Lasker	RM095 RM167
	25		W	(1642) Isaac Newton (1900) Antoni Zygmund	RM071
	26	T	(1780) Mary Fairfax Greig Somerville (1791) Charles Babbage (1937) John Horton Conway	RM059 RM119	
	27	F	(1571) Johannes Kepler (1654) Jacob (Jacques) Bernoulli	RM093	
	28	S	(1808) Louis Victoire Athanase Dupré (1882) Arthur Stanley Eddington (1903) John von Neumann	RM179 RM107	
	29	S	(1856) Thomas Jan Stieltjes		
	53	30	M	(1897) Stanislaw Saks	
		31	T	(1872) Volodymyr Levitsky (1896) Carl Ludwig Siegel (1945) Leonard Adleman (1952) Vaughan Frederick Randall Jones	RM143



Putnam 2009, B6

Prove that for every positive integer n , there is a sequence of integers $a_0, a_1, \dots, a_{2009}$ with $a_0=0$ and $a_{2009}=n$ such that each term after a_0 is either an earlier term plus 2^k for some nonnegative integer k , or of the form $b \bmod c$ for some earlier positive terms b and c .

[Here $b \bmod c$ denotes the remainder when b is divided by c , so $0 \leq (b \bmod c) < c$.]

How to put an elephant into a refrigerator

Statistics

Cut off the elephant's tail, place it in the refrigerator, and consider it a representative sample of the entire elephant.

Murphy's Laws applied to Math Students

Every problem is harder than it looks and takes longer than you expected.

Although for most of us [the law of universal gravitation] is of no practical use, we can all appreciate its elegant conciseness. A couple of small multiplications, a simple division and there you go: wherever you are, you can know your gravitational position.

Bill Bryson

It is true that Fourier was of the opinion that the main purpose of mathematics was public utility and the explanation of natural phenomena; but a philosopher like him should have known that the only purpose of science is to honor the human mind, and in this regard a question about number theory is as valid as a question about how the world is made.

Karl Gustav Jacob Jacobi

The ability to do mathematics has supported and fuelled trust in human reason, and at the same time rooted a superhuman vision of mathematics.

Gabriele Lolli

*Numero pondere et mensura Deus omnia condidit
God created everything by number, weight and measure.*

Isaac Newton

There are many questions fools can ask, that wise men cannot answer.

George Polya

Einstein: "You know, Henri, I used to study mathematics, but I left it for physics."

Poincaré: "Oh, really, Albert? But why?"

Einstein: "Because although I could distinguish true statements from false ones, I could not determine which facts were important."

Poincaré: "This is very interesting, Albert, because originally I studied physics, but I left it for mathematics."

Einstein: "Really? Why?"

Poincaré: "Because I was unable to say which of the important facts were true."

David Singmaster