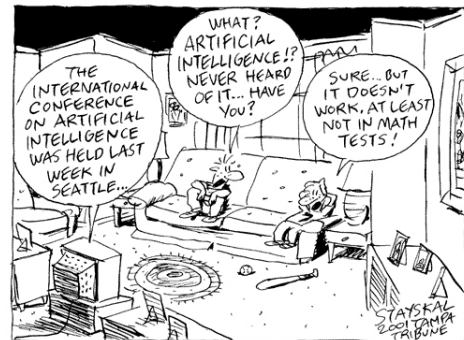
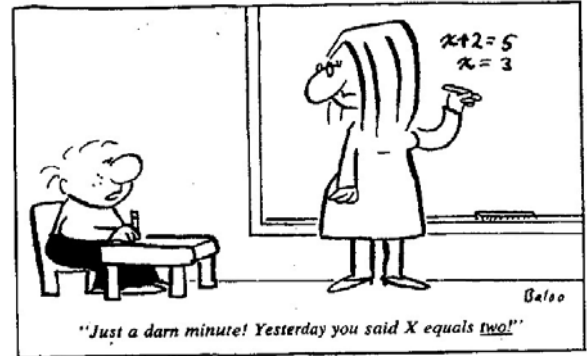
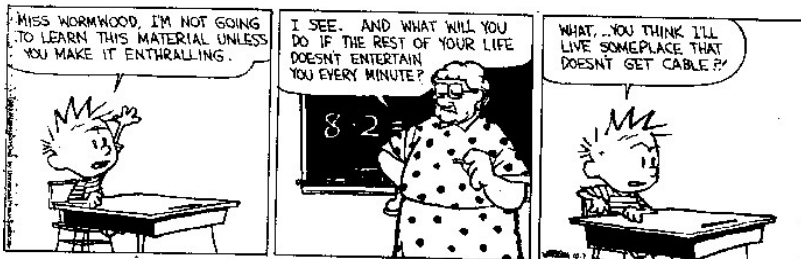


$$x^4 - 8180x^3 + 25090190x^2 - 34200948100x + 17481136677369 = 0$$

Sam and Silo by dumas



Calvin and Hobbes



1	1	W	(1803) Guglielmo LIBRI Carucci dalla Somaja (1878) Agner Krarup ERLANG (1894) Satyendranath BOSE (1912) Boris GNEDENKO
	2	T	(1822) Rudolf Julius Emmanuel CLAUSIUS (1905) Lev Genrichovich SHNIRELMAN (1938) Anatoly SAMOILENKO
	3	F	(1917) Yuri Alexeievich MITROPOLSHY
	4	S	(1643) Isaac NEWTON
	5	S	(1838) Marie Ennemond Camille JORDAN (1871) Federigo ENRIQUES (1871) Gino FANO
2	6	M	(1807) Jozeph Mitza PETZVAL (1841) Rudolf STURM
	7	T	(1871) Felix Edouard Justin Emile BOREL (1907) Raymond Edward Alan Christopher PALEY
	8	W	(1888) Richard COURANT (1924) Paul Moritz COHN (1942) Stephen William HAWKING
	9	T	(1864) Vladimir Adreievich STELKOV
	10	F	(1875) Issai SCHUR (1905) Ruth MOUFANG
	11	S	(1545) Guidobaldo DEL MONTE (1707) Vincenzo RICCATI (1734) Achille Pierre Dionis DU SEJOUR
	12	S	(1906) Kurt August HIRSCH
3	13	M	(1864) Wilhelm Karl Werner Otto Fritz Franz WIEN (1876) Luther Pfahler EISENHART (1876) Erhard SCHMIDT
	14	T	(1902) Alfred TARSKI
	15	W	(1704) Johann CASTILLON (1717) Matthew STEWART (1850) Sofia Vasilievna KOVALEVSKAJA
	16	T	(1801) Thomas KLAUSEN
	17	F	(1847) Nikolay Egorovich ZUKOWSKY (1858) Gabriel KOENIGS
	18	S	(1856) Luigi BIANCHI (1880) Paul EHRENFEST
	19	S	(1813) Rudolf Friedrich Alfred CLEBSCH (1879) Guido FUBINI (1908) Aleksandr Gennadjevich KUROK
4	20	M	(1775) Andre Marie AMPERE (1895) Gabor SZEGO (1904) Renato CACCIOPPOLI
	21	T	(1846) Pieter Hendrik SCHOÛTE (1915) Yuri Vladimirovich LINNIK
	22	W	(1592) Pierre GASSENDI (1908) Lev Davidovich LANDAU
	23	T	(1840) Ernst ABBE (1862) David HILBERT
	24	F	(1891) Abram Samoilovitch BESICOVITCH (1914) Vladimir Petrovich POTAPOV
	25	S	(1627) Robert BOYLE (1736) Joseph-Louis LAGRANGE (1843) Karl Herman Amandus SCHWARTZ
	26	S	(1799) Benoit Paul Emile CLAPEYRON
5	27	M	(1832) Charles Lutwidge DODGSON
	28	T	(1701) Charles Marie de LA CONDAMINE (1892) Carlo Emilio BONFERRONI
	29	W	(1817) William FERREL (1888) Sidney CHAPMAN
	30	T	(1619) Michelangelo RICCI
	31	F	(1715) Giovanni Francesco FAGNANO dei Toschi (1841) Samuel LOYD (1896) Sofia Alexandrovna JANOWSKAJA

## APMO 1989 [1]

Let  $x_1, x_2, \dots, x_n$  be positive real numbers,

and let  $S = \sum_{i=1}^n x_i$ .

Prove that

$$\prod_{i=1}^n (1 + x_i) \leq \sum_{i=0}^n \frac{S^i}{i!}$$

## The Dictionary

*Clearly:* I don't want to write down all the "in-between" steps.

The First Law of Applied Mathematics: All infinite series converge, and moreover converge to the first term.

*A mathematician's reputation rests on the number of bad proofs he has given.*

Abram BESICOVITCH

*Probabilities must be regarded as analogous to the measurements of physical magnitudes; that is to say, they can never be known exactly, but only within certain approximation.*

Emile BOREL

*I have no certainties, at most probabilities.*

Renato CACCIOPPOLI

*What I tell you three times is true.*

Charles DODGSON

*The proof of the Hilbert Basis Theorem is not mathematics: it is theology.*

Camille JORDAN

*Probabilities must be regarded as analogous to the measurement of physical magnitudes: they can never be known exactly, but only within certain approximation.*

Emile BOREL

*God not only plays dice. He also sometimes throws the dice where they cannot be seen.*

Stephen HAWKING

5	1	S	(1900) John Charles BURKILL	
	2	S	(1522) Lodovico FERRARI	
6	3	M	(1893) Gaston Maurice JULIA	
	4	T	(1905) Eric Christopher ZEEMAN	
	5	W	(1757) Jean Marie Constant DUHAMEL	
	6	T	(1612) Antoine ARNAULD (1695) Nicolaus (II) BERNOULLI	
	7	F	(1877) Godfried Harold HARDY (1883) Eric Temple BELL	
	8	S	(1700) Daniel BERNOULLI (1875) Francis Ysidro EDGEWORTH	
	9	S	(1775) Farkas Wolfgang BOLYAI (1907) Harold Scott MacDonald COXETER	
7	10	M	(1747) Aida YASUAKI	
	11	T	(1800) William Henry Fox TALBOT (1839) Josiah Willard GIBBS (1915) Richard Wesley HAMMING	
	12	W	(1914) Hanna CAEMMERER NEUMANN	
	13	T	(1805) Johann Peter Gustav Lejeune DIRICHLET	
	14	F	(1468) Johann WERNER (1849) Hermann HANKEL (1896) Edward Artur MILNE	
	15	S	(1564) Galileo GALILEI (1861) Alfred North WHITEHEAD (1946) Douglas HOFSTADTER	
	16	S	(1822) Francis GALTON (1853) Georigio RICCI-CURBASTRO (1903) Beniamino SEGRE	
	8	17	M	(1890) Sir Ronald Aymler FISHER (1891) Adolf Abraham Halevi FRAENKEL
		18	T	(1404) Leon Battista ALBERTI
19		W	(1473) Nicolaus COPERNICUS	
20		T	(1844) Ludwig BOLTZMANN	
21		F	(1591) Girard DESARGUES (1915) Evgenii Michailovitch LIFSHITZ	
22		S	(1903) Frank Plumpton RAMSEY	
23		S	(1583) Jean-Baptiste MORIN (1951) Shigefumi MORI	
9	24	M	(1871) Felix BERNSTEIN	
	25	T	(1827) Henry WATSON	
	26	W	(1786) Dominique Francois Jean ARAGO	
	27	T	(1881) Luitzen Egbertus Jan BROUWER	
	28	F	(1735) Alexandre Theophile VANDERMONDE (1860) Herman HOLLERITH	

### APMO 1989 [2]

Prove that the equation

$$6 * (6a^2 + 3b^2 + c^2) = 5n^2$$

has no solutions in integers except

$$a = b = c = n = 0$$

### The Dictionary

*Trivial:* If I have to show you how to do this, you're in the wrong class

There are two groups of people in the world: those who believe that the world can be divided into two groups of people, and those who don't.

*Connaitre, decouvrir, communiquer.*  
*Telle est la destinée d'un savant*

François ARAGO

*Common sense is not really so common*

Antoine ARNAULD

*"Obvious" is the most dangerous word in mathematics.*

Eric Temple BELL

*...it would be better for the true physics if there were no mathematicians on hearth.*

Daniel BERNOULLI

*...an incorrect theory, even if it cannot be inhibited bay any contradiction that would refute it, is none the less incorrect, just as a criminal policy is none the less criminal even if it cannot be inhibited by any court that would curb it.*

Jan BROUWER

*Mathemata mathematici scribuntur*

Nicolaus COPERNICUS

9	1	S	(1611) John PELL
	2	S	(1836) Julius WEINGARTEN
10	3	M	(1838) George William HILL (1845) Georg CANTOR
	4	T	(1822) Jules Antoine LISSAJUS
	5	W	(1512) Gerardus MERCATOR (1759) Benjamin GOMPertz (1817) Angelo GENOCCHI
	6	T	(1866) Ettore BORTOLOTTI
	7	F	(1792) William HERSCHEL (1824) Delfino CODAZZI
	8	S	(1851) George CHRYSAL
	9	S	(1818) Ferdinand JOACHIMSTHAL (1900) Howard Hathaway AIKEN
11	10	M	(1864) William Fogg OSGOOD
	11	T	(1811) Urbain Jean Joseph LE VERRIER (1853) Salvatore PINCHERLE
	12	W	(1685) George BERKELEY (1824) Gustav Robert KIRKHOFF (1859) Ernesto CESARO
	13	T	(1861) Jules Joseph DRACH (1957) Rudy D'ALEMBERT
	14	F	(1864) Jozef KURSCHAK (1879) Albert EINSTEIN
	15	S	(1860) Walter Frank Raphael WELDON (1868) Grace CHISOLM YOUNG
	16	S	(1750) Caroline HERSCHEL (1789) Georg Simon OHM (1846) Magnus Gosta MITTAG-LEFFLER
12	17	M	(1876) Ernest Benjamin ESCLANGON (1897) Charles FOX
	18	T	(1640) Philippe de LA HIRE (1690) Christian GOLDBACH (1796) Jacob STEINER
	19	W	(1862) Adolf KNESER (1910) Jacob WOLFOVITZ
	20	T	(1840) Franz MERTENS (1884) Philip FRANCK (1938) Sergi Petrovich NOVIKOV
	21	F	(1768) Jean Baptiste Joseph FOURIER (1884) George David BIRKHOFF
	22	S	(1917) Irving KAPLANSKY
	23	S	(1754) Georg Freiherr von VEGA (1882) Emmy Amalie NOETHER (1897) John Lighton SYNGE
13	24	M	(1809) Joseph LIOUVILLE (1948) Sun-Yung (Alice) CHANG
	25	T	(1538) Christopher CLAUDIUS
	26	W	(1848) Konstantin ADREEV (1913) Paul ERDOS
	27	T	(1857) Karl PEARSON
	28	F	(1749) Pierre Simon de LAPLACE
	29	S	(1825) Francesco FAA' DI BRUNO (1873) Tullio LEVI-CIVITA (1896) Wilhelm ACKERMAN
	30	S	(1892) Stefan BANACH
14	31	M	(1596) Rene` DESCARTES

### APMO 1989 [3]

Let  $A_1, A_2, A_3$  be three points in the plane, and for convenience let  $A_4=A_1, A_5=A_2$ . For  $n=1, 2,$  and  $3$  suppose that  $B_n$  is the midpoint of  $A_nA_{n+1}$ , and suppose that  $C_n$  is the midpoint of  $A_nB_n$ . Suppose that  $A_nC_{n+1}$  and  $B_nC_{n+2}$  meet at  $D_n$ , and that  $A_nB_{n+1}$  meet at  $E_n$ . Calculate the ratio of the area of triangle  $D_1D_2D_3$  to the area of triangle  $E_1E_2E_3$ .

### The Dictionary

*It can easily be shown:* No more than four hours are needed to prove it

Theorem: All the numbers are boring  
Proof (by contradiction): Suppose  $x$  is the first non-boring number. Who cares?

*Mathematics is the most beautiful and the most powerful creation of the human spirit. Mathematics is as old as Man.*

Stefan BANACH

*In mathematics the art of proposing a question must be held on higher value than solving it.*

Georg CANTOR

*When writing about transcendental issues, be transcendently clear.*

Rene` DESCARTES

*The search for truth is more important than its possession.*

Albert EINSTEIN

*Property is a nuisance.*

Paul ERDOS

*Don't worry about people stealing your ideas. If your ideas are any good, you'll have to ram them down people's throat.*

Howard AIKEN

*Geometry is the noblest branch of physics.*

William OSGOOD

14	1	T	(1640) Georg MOHR (1776) Marie-Sophie GERMAIN (1895) Alexander Craig AITKEN	
	2	W	(1934) Paul Joseph COHEN	
	3	T	(1835) John Howard Van AMRINGE (1892) Hans RADEMACHER (1900) Albert Edward INGHAM (1909) Stanislaw Marcin ULAM (1971) Alice RIDDLE	
	4	F	(1809) Benjamin PEIRCE (1842) Francois Edouard Anatole LUCAS (1949) Shing-Tung YAU	
	5	S	(1588) Thomas HOBBS (1607) Honore FABRI (1622) Vincenzo VIVIANI (1869) Sergei Alexeievich CHAPLYGIN	
	6	S		
15	7	M	(1768) Francois Joseph FRANCAIS	
	8	T	(1903) Marshall Harvey STONE	
	9	W	(1791) George PEACOCK (1816) Charles Eugene DELAUNAY (1919) John Presper HECKERT	
	10	T	(1857) Henry Ernest DUDENEY	
	11	F	(1953) Andrew John WILES	
	12	S	(1794) Germaine Pierre DANDELIN (1852) Carl Louis Ferdinand Von LINDEMANN (1903) Jan TINBERGEN	
	13	S	(1728) Paolo FRISI (1813) Duncan Farquharson GREGORY (1879) Francesco SEVERI	
16	14	M	(1629) Christiaan HUYGENS	
	15	T	(1452) Leonardo da VINCI (1548) Pietro Antonio CATALDI (1707) Leonhard EULER (1809) Herman Gunther GRASSMANN	
	16	W	(1682) John HADLEY (1823) Ferdinand Gotthold Max EISENSTEIN	
	17	T	(1798) Etienne BOBILLIER (1853) Arthur Moritz SCHONFLIES	
	18	F	(1907) Lars Valerian AHLFORS (1918) Hsien-Chung WANG (1949) Charles Louis FEFERMAN	
	19	S	(1880) Evgeny Evgenievich SLUTSKY (1883) Richard VIN MISES (1901) Kiyoshi OKA (1905) Charles EHRESMANN	
	20	S	(1839) Francesco SIACCI	
	17	21	M	(1652) Michel ROLLE (1774) Jean Baptiste BIOT (1875) Teiji TAKAGI
		22	T	(1811) Otto Ludwig HESSE (1887) Harald August BOHR
		23	W	(1858) Max Karl Ernst Ludwig PLANCK
24		T	(1863) Giovanni VAILATI	
25		F	(1849) Felix Christian KLEIN (1900) Wolfgang PAULI (1903) Andrei Nicolayevich KOLMOGOROV	
26		S	(1889) Ludwig Josef Johan WITTEGENSTEIN	
27		S	(1755) Marc-Antoine PARSEVAL des Chenes	
18		28	M	(1906) Kurt GODEL
	29	T	(1854) Jules Henri POINCARÉ	
	30	W	(1777) Johann Carl Friedrich GAUSS (1916) Claude Elwood SHANNON	

**APMO 1989 [4]**

Let  $S$  be a set consisting of  $m$  pairs  $(a, b)$  of positive integers with the property that  $1 \leq a < b \leq n$ . Show that there are at least

$$4m * \frac{m - \frac{n^2}{4}}{3n}$$

triples  $(a, b, c)$  such that  $(a, b)$ ,  $(a, c)$  and  $(b, c)$  belong to  $S$ .

**The Dictionary**

*Check for yourself:* This is the boring part of the proof, so you can do it on your own time

It is proven that celebration of birthdays is healthy. Statistics show that those people that celebrate the most birthdays become the oldest.

*How wonderful that we have met with a paradox. Now we have some hope of making progress.*

Niels BOHR

*The notion of a set is too vague for the continuum hypothesis to have a positive or negative answer*

Paul COHEN

*Any good idea can be stated in fifty words or less.*

Stanislaw ULAM

*Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate.*

Leonhard EULER

*If anybody says he can think about quantum problems without getting giddy, that only shows he has not understood the first thing about them.*

Max PLANCK

18	1	T	(1825) Johann Jacob BALMER
	2	F	(1860) D'Arcy Wentworth THOMPSON (1905) Kazimierz ZARANKIEWITZ
	3	S	(1842) Otto STOLZ (1860) Vito VOLTERRA
	4	S	(1845) William Kingdon CLIFFORD
19	5	M	(1833) Lazarus Emmanuel FUCHS (1897) Francesco Giacomo TRICOMI
	6	T	(1872) Willem DE SITTER (1906) Andre' VEIL
	7	W	(1926) Alexis Claude CLAIRAUT (1854) Giuseppe VERONESE (1881) Ebenezer CUNNINGHAM (1896) Pavel Sergieievich ALEXANDROV
	8	T	(1859) Johan Ludvig William Valdemar JENSEN
	9	F	(1746) Gaspard MONGE (1876) Gilbert Ames BLISS
	10	S	(1788) Augustin Jean FRESNEL (1847) William Karl Joseph KILLING (1958) Piotr Rizeriovich SILVERBRAHMS
	11	S	(1918) Richard Phillips FEYNMAN
	20	12	M
13		T	(1750) Lorenzo MASCHERONI
14		W	(1832) Rudolf Otto Sigismund LIPSCHITZ (1863) John Charles FIELDS
15		T	(1939) Brian HARTLEY
16		F	(1718) Maria Gaetana AGNESI (1821) Pafnuti Lvovi CHEBYSHEV
17		S	
18		S	(1850) Oliver HEAVISIDE (1892) Bertrand Arthur William RUSSELL
21		19	M
	20	T	(1861) Henry Seely WHITE
	21	W	(1471) Albrecht DURER (1792) Gustave Gaspard de CORIOLIS
	22	T	(1865) Alfred Cardew DIXON
	23	F	(1914) Lipa BERS
	24	S	
	25	S	(1838) Karl Mikhaelovich PETERSON
22	26	M	(1667) Abraham DE MOIVRE (1896) Yuri Dimitrievich SOKOLOV
	27	T	(1862) John Edward CAMPBELL
	28	W	(1676) Jacopo Francesco RICCATI (1710) Johann (II) BERNOULLI
	29	T	(1882) Harry BATEMAN
	30	F	(1814) Eugene Charles CATALAN
	31	S	(1926) John KEMENY

### APMO 1989 [4]

Determine all the function  $f$  from the reals to the reals for which:

- $f(x)$  is strictly increasing
- $f(x)+g(x)=2x$  for all real  $x$ .

where  $g(x)$  is the composition inverse function to  $f(x)$ .

(Note:  $f$  and  $g$  are said to be composition inverses if  $f(g(x))=x$  and  $g(f(x))=x$  for all real  $x$ ).

### The Dictionary

Hint: The hardest of several possible ways to do a proof

Statistics are like a bikini. What is revealed is interesting, what is concealed is crucial.

*You may always depend on it that algebra which cannot be translated into good English and sound common sense is bad algebra*

William CLIFFORD

*And since geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art...*

Albrecht DURER

*Where did we get Schrodinger equation from? It's not possible to derive it from anything you know. It came out of the mind of Schrodinger.*

Richard FEYNMAN

*Nature is not embarrassed by difficulties of analysis*

Augustin FRESNEL

22	1	S	(1796) Sadi Leonard Nicolas CARNOT (1851) Edward Bailey ELLIOTT (1899) Edward Charles TITCHMARSH
23	2	M	(1895) Tibor RADO
	3	T	(1659) David GREGORY
	4	W	(1809) John Henry PRATT
	5	T	(1814) Pierre LAurent WANTZEL (1819) John Couch ADAMS
	6	F	(1436) Johann Muller REGIOMONTANUS (1857) Aleksandr Michailovitch LYAPUNOV (1906) Max ZORN
	7	S	(1863) Edward Burr VAN VLECK
	8	S	(1625) Giovanni Domenico CASSINI (1858) Charlotte Angas SCOTT (1860) Alicia Boole STOTT
	24	9	M
10		T	(940) Mohammad ABU'L-WAFA Al-Buzjani (1887) Vladimir Ivanovich SMIRNOV
11		W	(1937) David Bryant MUMFORD
12		T	(1888) Zygmunt JANYSZEWSKI
13		F	(1831) James Clerk MAXWELL (1876) William Sealey GOSSET (Student) (1928) John Forbes NASH
14		S	(1736) Charles Augustin de COULOMB (1856) Andrei Andreyevich MARKOV (1903) Alonzo CHURCH
15		S	(1640) Bernard LAMY (1894) Nikolai Gregorievich CHEBOTARYOV
25		16	M
	17	T	(1898) Maurits Cornelius ESCHER
	18	W	(1858) Andrew Russell FORSYTH (1884) Charles Ernest WEATHERBURN
	19	T	(1623) Blaise PASCAL (1902) Wallace John ECKERT
	20	F	(1873) Alfred LOEWY
	21	S	(1781) Simeon Denis POISSON (1828) Giuseppe BRUNO
	22	S	(1860) Mario PIERI (1864) Hermann MINKOWSKY (1910) Konrad ZUSE
	26	23	M
24		T	(1880) Oswald VEBLEN
25		W	(1908) William Van Orman QUINE
26		T	(1824) William THOMPSON, Lord Kelvin (1918) Yudell Leo LUKE
27		F	(1806) Augustus DE MORGAN
28		S	(1875) Henri Leon LEBESGUE
29		S	(1888) Aleksandr Aleksandrovich FRIEDMANN
27	30	M	(1791) Felix SAVART

### APMO 1990 [1]

In the triangle  $ABC$ , let  $D, E, F$  be the midpoints of  $BC, AC, AB$  respectively and let  $G$  the centroid of the triangle.

For each value of the angle  $BAC$ , how many non-similar triangles are there in which  $AEGF$  is a cyclic quadrilateral?

### The Dictionary

*Brute Force:* Four special cases, three counting arguments and two long inductions

What's the question the Cauchy distribution hates the most?

*Got a moment?*

*The sciences are like a beautiful river, of which the course is easy to follow when it has acquired a certain regularity; but if one wants to go back to the source, one will find it nowhere, because it is everywhere; it is spread so much [as to be] over all the surface of the earth; it is the same if one want to go back to the origin of the sciences, one will find only obscurity, vague ideas, vicious circles; and one loses oneself in the primitive ideas.*

Sadi CARNOT

*It is easier to square the circle than to get round a mathematician*

Augustus DE MORGAN

*Algebra goes to the heart of the matter at it ignores the casual nature of particular cases.*

Edward TITCHMARSH

*A good mathematical joke is better mathematics than a dozen of mediocre papers.*

John LITTLEWOOD

27	1	T	(1643) Gottfried Wilhelm von LEIBNITZ (1788) Jean Victor PONCELET	
	2	W	(1820) William John Racquorn RANKINE (1852) William BURNSIDE	
	3	T	(1807) Ernest Jean Philippe Fauque de JONQUIERE (1897) Jesse DOUGLAS	
	4	F	(1906) Daniel Edwin RUTHERFORD (1917) Michail Samuilovich LIVSIC	
	5	S		
	6	S	(1849) Alfred Bray KEMPE	
28	7	M	(1816) Johann Rudolf WOLF (1906) William FELLER (1922) Vladimir Aleksandrovich MARCHENKO	
	8	T	(1760) Christian KRAMP	
	9	W	(1845) George Howard DARWIN	
	10	T	(1862) Roger COTES (1868) Oliver Dimon KELLOGG	
	11	F	(1857) Sir Joseph LARMOR (1890) Giacomo ALBANESE	
	12	S	(1875) Ernest Sigismund FISCHER (1895) Richard BUCKMINSTER FULLER	
	13	S	(1527) John DEE (1741) Karl Friedrich HINDENBURG	
29	14	M		
	15	T	(1865) Wilhelm WIRTINGER (1906) Adolph Andrej Pavlovich YUSHKEVICH	
	16	W	(1678) Jakob HERMANN (1903) Irmgard FLUGGE-LOTZ	
	17	T	(1831) Victor Mayer Amedee MANNHEIM (1837) Wilhelm LEXIS	
	18	F	(1013) Hermann von REICHENAU (1635) Robert HOOKE (1853) Hendrich Antoon LORENTZ	
	19	S	(1768) Francois Joseph SERVOIS	
	20	S		
	30	21	M	(1620) Jean PICARD (1848) Emil WEYR (1849) Robert Simpson WOODWARD
		22	T	(1784) Friedrich Wilhelm BESSEL
		23	W	(1775) Etienne Louis MALUS (1854) Ivan SLEZYNSKY
24		T	(1851) Friedrich Herman SCHOTPKY (1871) Paul EPSTEIN (1923) Christine Mary HAMILL	
25		F	(1808) Johann Benedict LISTING	
26		S	(1903) Kurt MAHLER	
27		S	(1667) Johann BERNOULLI (1801) George Biddel AIRY (1848) Lorand Baron von EOTVOS (1871) Ernst Friedrich Ferdinand ZERMELO	
31		28	M	(1954) Gerd FALTINGS
		29	T	
	30	W		
	31	T	(1704) Gabriel CRAMER (1712) Johann Samuel KOENIG	

APMO 1990 [2]

Let  $a_1, a_2, \dots, a_n$  be positive real numbers, and let  $s_k$  be the sum of products of  $a_1, a_2, \dots, a_n$  taken  $k$  at a time. Show that, for  $k=1, 2, \dots, n-1$ :

$$s_k s_{n-k} \geq \binom{n}{k}^2 a_1 a_2 \cdots a_n$$

The Dictionary

*Elegant proof:* Requires no previous knowledge of the subject matter and is less than ten lines long

Arguing with a statistician is a lot like wrestling with a pig. After a few hours you begin to realize the pig likes it.

*All possible definitions of probability fall short of the actual practice.*

William FELLER

*I am a passenger of the spaceship Hearth.*

Richard BUCKMINSTER FULLER

*The imaginary number is a fine and wonderful resource of the human spirit, almost an amphibian between being and not being*

Gottfried LEIBNIZ

*A quantity that is increased or decreased of an infinitely small quantity is neither increased nor decreased*

Johann BERNOULLI

*...The science of Nature has already been too long made only a work of the brain and the fancy. It is now high time that it should return to the plainness and soundness of observations on material and obvious things.*

Robert HOOKE



31	1	F	(1861) Ivar Otto BENDIXSON (1881) Otto TOEPLITZ
	2	S	(1856) Ferdinand RUDIO (1902) Mina Spiegel REES
	3	S	(1914) Mark KAC
32	4	M	(1805) Sir William Rowan HAMILTON (1838) John VENN
	5	T	(1802) Niels Henrik ABEL
	6	W	(1638) Nicolas MALEBRANCHE (1741) John WILSON
	7	T	(1868) Ladislaus Josephowitsch BORTKIEWITZ
	8	F	(1902) Paul Adrien Maurice DIRAC
	9	S	(1537) Francesco BAROZZI (Franciscus Barocius)
	10	S	(1602) Gilles Personne de ROBERVAL
33	11	M	(1730) Charles BOSSUT (1842) Enrico D'OVIDIO
	12	T	(1882) Jules Antoine RICHARD (1887) Erwin Rudolf Josef Alexander SCHRODINGER
	13	W	(1625) Erasmus BARTHOLIN (1819) George Gabriel STOKES (1861) Cesare BURALI-FORTI
	14	T	(1530) Giovanni Battista BENEDETTI (1842) Jean Gaston DARBOUX (1865) Guido CASTELNUOVO (1866) Charles Gustave Nicolas de la VALLEE` POUSSIN
	15	F	(1863) Aleksei Nikolaevich KRYLOV (1892) Louis Pierre Victor duc de BROGLIE (1901) Petr Sergeevich NOVIKOV
	16	S	(12773) Louis Benjamin FRANCOEUR (1821) Arthur CAYLEY
	17	S	(1601) Pierre de FERMAT
34	18	M	(1685) Brook TAYLOR
	19	T	(1646) John FLAMSTEED (1739) Georg Simon KLUGEL
	20	W	(1710) Thomas SIMPSON (1863) Corrado SEGRE (1882) Waclaw SIERPINSKI
	21	T	(1789) Augustin Louis CAUCHY
	22	F	(1647) Denis PAPIN
	23	S	(1683) Giovanni POLENT (1829) Moritz Benedikt CANTOR
	24	S	(1561) Bartholomeo PITISCUS (1942) Karen Keskulla UHLENBECK
35	25	M	(1561) Philip van LANSBERGE (1844) Thomas MUIR
	26	T	(1728) Johann Heinrich LAMBERT (1875) Giuseppe VITALI
	27	W	(1858) Giuseppe PEANO
	28	T	(1796) Irene Jules BIENAYME`
	29	F	(1904) Leonard ROTH
	30	S	(1856) Carle David Tolme` RUNGE (1906) Olga TAUSSKY-TODD
	31	S	(1821) Hermann Ludwig Ferdinand von HELMHOLTZ

### APMO 1990 [3]

Consider all the triangles  $ABC$  which have a fixed base  $AB$  and whose altitude from  $C$  is a constant  $h$ .

For which of these triangles is the product of its altitudes a maximum?

### The Dictionary

*Similarly:* At least one line of the proof of this case is the same as before

A lottery is a tax on people who don't understand statistics.

*The divergent series are the invention of the devil.*

Niels ABEL

*Two seemingly incompatible conceptions can each represent an aspect of the truth. They may serve in turn to represent the facts without ever entering into direct conflict.*

Louis DE BROGLIE

*As for everything else, so for a mathematical theory: beauty can be perceived but not explained*

Arthur CAYLEY

*This result is too beautiful to be false: it is more important to have beauty in one's equations than to have them fit experiment.*

Paul DIRAC

*And perhaps, posterity will thank me for having shown it that the ancients did not know everything.*

Pierre de FERMAT

*There are surely worse things than being wrong, and being dull and pedantic are surely among them.*

Mark KAC

36	1	M	(1659) Joseph SAURIN (1835) William Stankey JEVONS
	2	T	(1878) Maurice Rene' FRECHET (1923) Rene' THOM
	3	W	(1814) James Joseph SYLVESTER (1884) Solomon LEFSCHETZ (1908) Lev Semenovich PONTRYAGIN
	4	T	(1809) Luigi Federico MENABREA
	5	F	(1667) Giovanni Girolamo SACCHERI (1725) Jean Etienne MONTUCLA
	6	S	(1859) Boris Jakovlevich BUKREEV (1863) Dimitri Aleksandrovich GRAVE
	7	S	(1707) George Louis Leclerc comte de BUFFON (1955) Efim ZELMANOV
37	8	M	(1584) Gregorius SAINT-VINCENT (1588) Marin MERSENNE
	9	T	(1860) Frank MORLEY
	10	W	(1839) Charles Sanders PEIRCE
	11	T	(1623) Stefano degli ANGELI (1877) sir James Hopwood JEANS
	12	F	(1891) Antoine Andre' Louis REYNAUD (1900) Haskell Brooks CURRY
	13	S	(1873) Constantin CARATHEODORY (1885) Wilhelm Johann Eugen BLASCHKE
	14	S	(1858) Henry Burchard FINE (1891) Ivan Matveevich VINOGRADOV
38	15	M	(973) Abu Arrayhan Muhammad ibn Ahmad AL BIRUNI (1886) Paul Pierre LEVY
	16	T	(1494) Francisco MAUROLICO (1736) Johann Nikolaus TETENS
	17	W	(1743) Marie Jean Antoine Nicolas de Caritat de CONDORCET (1826) Georg Friedrich Bernhard RIEMANN
	18	T	(1752) Adrien Marie LEGENDRE
	19	F	(1749) Jean Baptiste DELAMBRE
	20	S	(1842) Alexander Wilhelm von BRILL (1861) Frank Nelson COLÉ
	21	S	(1899) Juliusz Pawel SCHAUDER
39	22	M	(1765) Paolo RUFFINI (1769) Louis PUISSANT (1803) Jaques Charles Francois STURM
	23	T	(1768) William WALLACE (1900) David van DANTZIG
	24	W	(1501) Girolamo CARDANO (1625) Johan DE WITT (1801) Michail Vasilevich OSTROGRADSKI
	25	T	(1819) George SALMON (1888) Stefan MAZURKIEWICZ
	26	F	(1688) Willem Jakob 's GRAVESANDE (1854) Percy Alexander MACMAHON (1891) Hans REICHENBACH
	27	S	(1855) Paul Emile APPEL (1876) Earle Raymond HEDRICK (1919) James Hardy WILKINSON
	28	S	(1698) Pierre Louis Moreau de MAUPERTUIS (1761) Ferdinand Francois Desire' Budan de BOISLAURENT (1873) Julian Lowell COOLIDGE
	40	29	M
30		T	(1775) Robert ADRAIN (1829) Joseph WOLSTENHOLME (1883) Ernst HELLINGER

### APMO 1990 [5]

Show that for every integer  $n \geq 6$ , there exists a convex hexagon which can be dissected into exactly  $n$  congruent triangles

### The Dictionary

*Two line proof:* I'll leave out everything but the conclusion, you can't question 'em if you can't see 'em

A statistician is someone who is skilled at drawing a precise line from an unwarranted assumption to a foregone conclusion.

*The attempt to apply rational arithmetic to a problem in geometry resulted in the first crisis in the history of mathematics. The two relatively simple problems (the determination of the diagonal of a square and that of the circumference of a circle) revealed the existence of new mathematical beings for which no place could be found in the rational domain.*

David DANTZIG

*[de Prony's Tables] will not serve in the usual cases, but only in the extraordinary cases.*

Jean DELAMBRE

*I believe that proving is not a natural activity for mathematicians.*

Rene' THOM

*The early study of Euclid make me a hater of geometry.*

James SYLVESTER

*If it's just turning the crank it's algebra, but if it's got an idea on it it's topology.*

Solomon LEFSCHETZ

*For the intrinsic evidence of his creation, the Great Architect of the Universe now begins to appear as a pure mathematician.*

James JEANS

40	1	W	(1671) Luigi Guido GRANDI (1898) Bela KEREKJARTO	
	2	T	(1825) John James WALKER (1908) Arthur ERDELYI	
	3	F	(1944) Pierre Rene' DELIGNE	
	4	S	(1759) Louis Francois Antoine ARBOGAST (1797) Jerome SAVARY	
	5	S	(1732) Nevil MASKELYNE (1781) Bernhard Placidus Johann Nepomuk BOLZANO (1861) Thomas Little HEATH	
41	6	M	(1552) Matteo RICCI (1831) Julius Wilhelm Richard DEDEKIND (1908) Sergei Lvovich SOBOLEV	
	7	T	(1885) Niels BOHR	
	8	W	(1908) Hans Arnold HEILBRONN	
	9	T	(1581) Claude Gaspard BACHET de Meziriac (1704) Johann Andrea von SEGNER (1873) Karl SCHWARTZSCHILD	
	10	F	(1861) Heinrich Friedrich Karl Ludwig BURKHARDT	
	11	S	(1675) Samuel CLARKE (1777) Barnabe BRISSON (1865) Alfred HAAR (1910) Cahit ARF	
	12	S	(1860) Elmer SPERRY	
	42	13	M	(1890) Georg FEIGL (1893) Kurt Werner Friedrich REIDEMEISTER (1932) John Griggs THOMSON
		14	T	(1687) Robert SIMSON (1801) Joseph Antoine Ferdinand PLATEAU (1868) Alessandro PADQA
		15	W	(1608) Evangelista TORRICELLI (1735) Jesse RAMSDEN (1776) Peter BARLOW
		16	T	(1879) Philip Edward Bertrand JOURDAIN
		17	F	(1759) Jacob (II) BERNOULLI (1888) Paul Isaac BERNAYS
18		S	(1741) John WILSON	
19		S	(1903) Jean Frederic Auguste DELSARTE (1910) Subrahmanyan CHANDRASEKHAR	
43		20	M	(1632) Sir Christopher WREN (1863) William Henry YOUNG (1865) Aleksandr Petrovich KOTELNIKOV
		21	T	(1677) Nicolaus (I) BERNOULLI (1823) Enrico BETTI (1855) Giovan Battista GUCCIA (1893) William LEonard FERRAR
	22	W	(1587) Joachim JUNGLIUS (1895) Rolf Herman NEVANLINNA (1907) Sarvadaman CHOWLA	
	23	T	(1865) Piers BOHL	
	24	F	(1804) Wilhelm Eduard WEBER (1873) Edmund Taylor WITAKER	
	25	S	(1811) Evariste GALOIS	
	26	S	(1849) Ferdinand Georg FROBENIUS (1857) Charles Max MASON (1911) Shiing-Shen CHERN	
	44	27	M	(1678) Pierre Remond de MONTMORT (1856) Ernest William HOBSON
28		T	(1804) Pierre Francois VERHULST	
29		W	(1925) Klaus ROTH	
30		T	(1906) Andrej Nikolaevich TIKHONOV	
31		F	(1815) Karl Theodor Wilhelm WEIERSTRASS	

**APMO 1991 [1]**

Let  $G$  be the centroid of the triangle  $ABC$  and  $M$  be the midpoint of  $BC$ . Let  $X$  be on  $AB$  and  $Y$  on  $AC$  such that the point  $X$ ,  $Y$ , and  $G$  are collinear and  $XY$  and  $BC$  are parallel. Suppose that  $XC$  and  $GB$  intersect at  $Q$  and  $YB$  and  $GC$  intersect at  $P$ . Show that triangle  $MPQ$  is similar to triangle  $ABC$ .

**The Dictionary**

*Briefly:* I'm running out of time, so I'll just write and talk faster

A statistician always *has something to say* with numbers while a politician *has to say something* with numbers.

$2^{30}(2^{31}-1)$  is the greatest perfect number that will ever be discovered, for, as they are merely curious without being useful, it is not likely that any person will attempt to find a number beyond it.

Peter BARLOW

As professor in the Polytechnic School in Zurich I found myself for the first time obliged to lecture upon the elements of the differential calculus and felt more keenly than ever before the lack of a really scientific foundation of the arithmetic.

Richard DEDEKIND

Prediction is very difficult, especially about the future.

Niels BOHR

Newton is, of course, the greatest of all Cambridge professors; he also happens to be the greatest disaster that ever befell not merely Cambridge mathematics in particular, but British mathematical science as a whole.

Leonard ROTH

A mathematician who is not also something of a poet will never be a perfect mathematician

Karl WEIERSTRASS

44	1	S	(1535) Giambattista DELLA PORTA	
	2	S	(1815) George BOOLE	
45	3	M	(1867) Martin Wilhelm KUTTA (1878) Arthur Byron COBLE	
	4	T	(1744) Johann (III) BERNOULLI (1865) Pierre Simon GIRARD	
	5	W	(1848) James Whitbread Lee GLAISHER (1930) John Frank ADAMS	
	6	T	(1781) Giovanni Antonio Amedeo PLANA	
	7	F	(1660) Thomas Fantet DE LAGNY (1799) Karl Heinrich GRAFFE (1898) Raphael SALEM	
	8	S	(1656) Edmond HALLEY (1846) Eugenio BERTINI (1848) Friedrich Ludwig Gottlob FREGE (1854) Johannes Robert RYDBERG (1869) Felix HAUSDORFF	
	9	S	(1847) Carlo Alberto CASTIGLIANO (1885) Théodor Franz Eduard KALUZA (1885) Hermann Klaus Hugo WEYL (1906) Jaroslav Borisovich LOPATYNSKY (1922) Imre LAKATOS	
	46	10	M	(1829) Helwin Bruno CHRISTOFFEL
		11	T	(1904) John Henry Constantine WHITEHEAD
12		W	(1825) Michail Egorovich VASHCHENKO-ZAKHARCHENKO (1842) John William STRUTT Lord RAYLEIGH (1927) Yutaka TANIYAMA	
13		T	(1876) Ernest Julius WILKZYNSKY (1878) Max Wilhelm DEHN	
14		F	(1845) Ulisse DINI	
15		S	(1688) Louis Bertrand CASTEL (1793) Michel CHASLES (1794) Franz Adolph TAURINUS	
16		S	(1835) Eugenio BELTRAMI	
47		17	M	(1597) Henry GELLIBRAND (1717) Jean Le Rond D'ALEMBERT (1790) August Ferdinand MOBIUS
		18	T	(1872) Giovanni Enrico Eugenio VACCA (1927) Jon Leslie BRITTON
	19	W	(1894) Heinz HOPF (1900) Michail Alekseevich LAVRENTEV (1901) Nina Karlovna BARI	
	20	T	(1889) Edwin Powell HUBBLE (1924) Benoit MANDELBROT	
	21	F	(1867) Dimitri SINTSOV	
	22	S	(1803) Giusto BELLAVIDIS (1840) Emile Michel Hyacinte LEMOINE	
	23	S	(1616) John WALLIS (1620) Issac TODHUNTER	
	48	24	M	(1549) Duncan MacLaren Young SOMERVILLE (1909) Gerhard GENTZEN
		25	T	(1873) Claude Louis MATHIEU (1841) Fredrich Wilhelm Karl Ernst SCHRODER
26		W	(1894) Norbert WIENER (1946) Enrico BOMBIERI	
27		T	(1867) Arthur Lee DIXON	
28		F	(1898) John WISHART	
29		S	(1803) Christian Andreas DOPPLER (1849) Horace LAMB (1879) Nikolay Mitrofanovich KRYLOV	
30		S	(1549) Sir Henry SAVILE	

**APMO 1991 [3]**

Let  $a_1, a_2, \dots, a_n$  and  $b_1, b_2, \dots, b_n$  be positive and

real numbers such that  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$ .

Show that:

$$\sum_{i=1}^n \frac{a_i^2}{a_i + b_i} \geq \frac{\sum_{i=1}^n a_i}{2}$$

**The Dictionary**

*Proceed formally:* Manipulate symbols by the rules without any hint of their true meaning

Statisticians are like the drunk leaning against the lamp pole: they are there for support, not illumination.

*Of the many forms of false culture, a premature converse with abstractions is perhaps the most likely to prove fatal to the growth of a masculine vigour of intellect.*

George BOOLE

*Algebra is generous: she often gives more than is asked for*

Jean D'ALEMBERT

*Mathematics is the only instructional material that can be presented in an entirely undogmatic way*

Max DEHN

*A scientist can hardly meet with anything more undesirable than to have the foundations give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through in press.*

Gottlob FREGE

*A modern mathematical proof is not very different from a modern machine: the simple fundamental principles are hidden under a mass of technical details.*

Hermann WEYL

49	1	M	(1792) Nikolay Yvanovich LOBACHEVSKY
	2	T	(1831) Paul David Gustav DU BOIS-RAYMOND (1901) George Frederick James TEMPLE
	3	W	(1903) Sidney GOLDSTEIN (1924) John BACKUS
	4	T	(1795) Thomas CARLYLE
	5	F	(1868) Arnold Johannes Wilhelm SOMMERFELD (1901) Werner Karl HEISENBERG
	6	S	(1682) Giulio Carlo FAGNANO dei Toschi
	7	S	(1647) Giovanni CEVA (1823) Leopold KRONECKER (1830) Antonio Luigi Gaudenzio Giuseppe CREMONA
50	8	M	(1508) Regnier GEMMA FRISIUS (1865) Jaques Salomon HADAMARD (1919) Julia Bowman ROBINSON
	9	T	(1883) Nikolai Nikolaievich LUZIN (1906) Grace Brewster MURRAY HOPPER (1917) Sergei Vasilovich FOMIN
	10	W	(1804) Karl Gustav Jacob JACOBI (1815) Augusta Ada KING Countess of LOVEFACE
	11	T	(1882) Max BORN
	12	F	(1832) Peter Ludwig Mejdell SYLOW
	13	S	(1724) Franz Ulrich Theodosius AEPINUS (1887) George POLYA
	14	S	(1546) Tycho BRAHE
51	15	M	(1802) Janos BOLYAI
	16	T	(1804) Wiktor Yakovievich BUNYAKOWSKY
	17	W	(1706) Gabrielle Emile Le Tonnelier de Breteuil du CHATELET (1835) Felice CASORATI (1842) Marius Sophus LIE (1900) Dame Mary Lucy CARTWRIGHT
	18	T	(1917) Roger LYNDON
	19	F	(1783) Charles Julien BRIANCHON (1854) Marcel Louis BRILLOUIN
	20	S	(1494) Oronce FINE (1648) Tommaso CEVA (1875) Francesco Paolo CANTELLI
	21	S	(1878) Jan LUKASIEVIKZ (1932) John Robert RINGROSE
52	22	M	(1824) Francesco BRIOSCHI (1859) Otto Ludwig HOLDER (1877) Tommaso BOGGIO (1887) Srinivasa Aiyangar RAMANUJAN
	23	T	(1872) Georgii Yurii PFEIFFER
	24	W	(1822) Charles HERMITE (1868) Emmanuel LASKER
	25	T	(1642) Isaac NEWTON (1900) Antoni ZYGMUND
	26	F	(1780) Mary Fairfax Greig SOMERVILLE (1791) Charles BABBAGE
	27	S	(1571) Johannes KEPLER (1654) Jacob (Jacques) BERNOULLI
	28	S	(1808) Athanase Louis Victoire DUPRE (1882) Arthur Stanley EDDINGTON (1903) John von NEUMANN
53	29	M	(1856) Thomas Jan STIELTJES
	30	T	(1897) Stanislaw SAKS
	31	W	(1872) Volodymyr LEVIYTSKY (1896) Carl Ludwig SIEGEL (1952) Vaughan Frederick Randall JONES

**APMO 1991 [5]**

Given are two tangent circles and a point  $P$  on their common tangent perpendicular to lines joining their centres. Construct with ruler and compass all the circles that are tangent to these two circles and pass through the point  $P$ .

**The Dictionary**

*Proof omitted:* Trust me, It's true

*Life, Math & ∇*

- Life is complex. It has real and imaginary components.
- To a Mathematician, real life is a special case.

*Errors using inadequate data are much less than those using no data at all.*

Charles BABBAGE

*Out of nothing I have created a strange new universe.*

Janos BOLYAI

*I am now convinced that theoretical physics is actual philosophy.*

Max BORN

*It is a mathematical fact that the casting of this pebble from my hand alters the centre of gravity of the universe*

Thomas CARLYLE

*Shuffling is the only thing that Nature cannot undo.*

Arthur EDDINGTON

*Number theorists are like lotus eaters: having tasted this food they can never give it up.*

Leopold KRONECKER

*Logic merely sanctions the conquests of intuition.*

Jaques HADAMARD

*If there is a problem you can't solve, then there is an easier problem you can't solve: find it.*

George POLYA